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Comparing Kalman Filter and Diffuse Kalman Filter on a GPS Signal with Noise

Maximo Giovani Tandazo Espinoza*

Universidad Politécnica Salesiana, Computer Science, Guayaquil, Ecuador

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ABSTRACT

The navigation control of an autonomous vehicle can be determined by the coordinates of a GPS (Global Positioning System) positioning system, angular velocity, and acceleration with an INS (Inertial Navigation System). However, the errors associated with these devices do not allow it to be the only measurement system used in an autonomous vehicle. The need arises to implement tools that determine the system's state reliably at any instant and perform the necessary control actions to fulfill the trajectory optimally, considering the system's internal model. Therefore, applying a Diffuse Kalman filter is vital, allowing information integration from GPS and other devices. This work was divided into three essential parts such as the Kalman filter, the fuzzy control, and the simulation of a GPS sensor signal, taking into account that, in this last part, a comparison is made with the behavior of a Diffuse Kalman filter. In general, due to the comparisons of the position estimations in GPS measurements, it is evident that the DKF achieves more efficient reliability values since the position estimation error is reduced.

1. Introduction

The navigation control of an autonomous vehicle can be determined by the coordinates of a GPS (Global Positioning System), angular velocity, and acceleration with an INS (Inertial Navigation System). However, the errors associated with these devices do not allow it to be the only measurement system used in an autonomous vehicle. The need arises then to implement tools that will enable the system to reliably determine the state of the system at any instant and perform the necessary control actions to fulfill the trajectory optimally, considering the system's internal model. Therefore, applying a Diffuse Kalman filter (DKF) is critical at this stage since it allows information integration from GPS and other devices. This work was divided into 3 essential parts: the Kalman filter (KF), the fuzzy control, and the simulation of a GPS sensor signal, considering that in this last part, a comparison is made with the behavior of a Diffuse Kalman filter. With the position estimation in GPS measurements, high-reliability values are achieved. These variations depend very much on how the position measurements are taken to model the noise to which it is exposed. One of the advantages of the Kalman filter is that it avoids acquiring very accurate sensors since, if sensors with more accurate clocks and more advanced position measurement techniques were considered, the costs would rise considerably.

The Kalman filter originates in the paper "A New Approach to Linear Filtering and Prediction Problems" published by Rudolf Emil Kalman in 1960, where he describes a recursive solution to the problem of linear filtering of discrete data [1]. Nowadays, it is a widely used method to optimally estimate the states of a dynamic system in real-time from the indirect noisy measurements that are taken from it [2]. These real-time estimates of the system state are valuable when operating in the open loop; however, if closed-loop operation is considered, they can be used to control and keep the vehicle in the desired direction.

Filtering is desirable in engineering and embedded systems situations since a good filtering algorithm can eliminate noise from the various eventualities to which the process to be monitored is exposed. There are likely fluctuations or disturbances caused by the environment or the existing sensor(s) characteristics. The Kalman filter is a method for estimating the variables of various processes. The Kalman filter estimates the states of a linear system in mathematical terms. The filter is an algorithm based on the state space model of a plan to assess the future shape and output by optimally filtering the output signal, and depending on the delay of the incoming samples, it can serve as an estimator or a filter but in both cases, it can eliminate noise. The filter applied to a model formulated in the state space allows a unified treatment of various aspects, such as the estimation of model parameters, the prediction of values, or the analysis of the system dynamics. This paper presents a review of the principal associated ideas. Among its applications are demographic

*Corresponding Author: Maximo Giovani Tandazo Espinoza, Universidad Politécnica Salesiana, Computer Science, Guayaquil, Ecuador, Email: mtandazo@ups.edu.ec

estimation, signal processing, navigation systems, predicting the behavior of economic variables, and image processing, among others. Due to its wide field of action, it is essential to know how it works to have the basic tools that solve several practical problems simply and optimally [3].

2. Materials and Methods

2.1. The Kalman Filter

Rudolf Emil Kalman's 1960 work "A New Approach to Linear Filtering and Prediction Problems" contains the original description of the Kalman filter. It outlines a recursive solution to the linear filtering of discrete data problems. Recursive least squares estimate is at the heart of state-space models, which encompass the work he did [1].

The Kalman filter is an ideal state estimation method for dynamic systems with stochastic disturbances. More specifically, the Kalman filter provides a linear recursive algorithm that estimates the states of a dynamic system with discrete noise in real-time, ideally and with minimal variance error. Radar, ballistic missile trajectory calculation, satellite navigation, and video and laser tracking systems are just a few of the many industrial, civil, and military uses for it. Real-time applications of the Kalman filter have been developed in response to the swift advancement of faster computers [4]. The Kalman filter is also seen as an effective and versatile procedure to combine the output of sensors with noise to estimate the states of a system with uncertain dynamics.

In essence, this filter is a set of mathematical formulas that construct an optimal predictor-corrector type estimator, meaning that given certain conditions, it minimizes the estimated covariance error [5]. A variety of time series models can be handled with the state-space representation, which is a helpful notation for estimating stochastic models where mistakes in the system's measurement are assumed. Specific applications include the depiction of other models that need maximum likelihood approximation and the modeling of time-varying parameters and unobservable components. The filter is a mathematical process that uses a mechanism for correction and prediction. This algorithm predicts the next state based on its previous estimation by introducing a correction term proportionate to the prediction error and minimizing it statistically [5, 6, 7, 8].

The Kalman filter is the most popular linear estimation method for estimating the states and parameters of linear and nonlinear dynamic systems. Using a mathematical model of the plant and a measurement model of the sensor systems [9], the Kalman filter can anticipate the future values of any dynamic system. In attempting to collect the most accurate estimates of the system state, inaccurate data still frequently appears. According to [10], the user's driving style can alter the battery's state of charge (SOC), which is influenced by several factors, including temperature, internal resistance, capacity, and charge and discharge rates, among others. The Kalman filter is used to remove erroneous data due to these causes. The process consists of calculating the new state and its uncertainties.

Within the state-space notation, the derivation of the Kalman filter rests on the assumption of normality of the initial state vector and system perturbations in such a way that it is possible to calculate the likelihood function on the prediction error with which the estimation of the unknown parameters of the system is carried out (see figure 1).

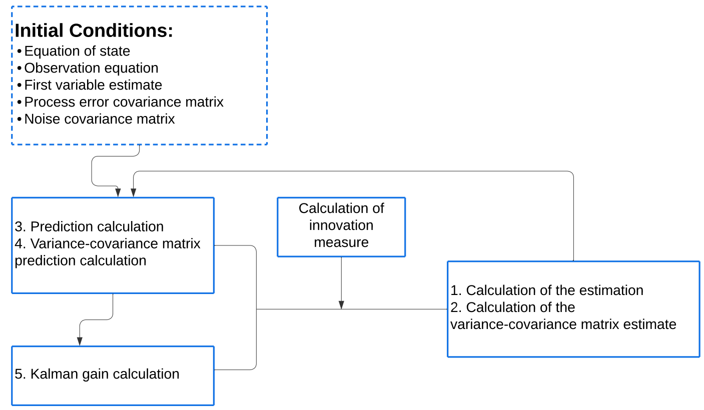

Figure 1: Kalman Algorithm Summary Diagram

2.2. Process estimation

The filter aims to solve the general problem of estimating the state $X \in R^n$ of a discrete-time controlled process, which is governed by the linear equation of the stochastic differential equation as follows:

$$X_k = Ax_{k-1} + Bu_k + w_{k-1} \quad (1)$$

With a measurement $z \in R^m$ that is:

$$Z_k = Hx_k + v_k \quad (2)$$

The random variables w_k and v_k represent the process and measurement error respectively. They are assumed to be independent of each other, white noise, and with normal probability distribution:

$$P(w) \approx N(0, Q) \quad (3)$$

$$P(v) \approx N(0, R) \quad (4)$$

Although they can be considered constant, the process noise covariance Q and the measurement noise covariance R are matrices that could vary with each time step or measurement in practice. Matrix A of size n x n relates the state at the previous time step k - 1 to that at the current time step k. Matrix B of size n x l relates the optional control input $u \in R^l$ to the state x. Matrix H of size m x n relates the state to the measurement z_k. In practice, these matrices may change each time, but they are usually assumed to be constant.

The KF estimates the process mentioned above using a feedback control. In other words, it evaluates the process at a particular moment and then uses the observed data to get feedback. Two sets of equations can be employed to obtain the filter: one updates the time or prediction equations, and the other updates the observed data or update equations. Those of the first group are responsible for the state projection at time k, taking as reference the state at time k-1, and for the intermediate update of the state covariance matrix. The second group of equations is responsible for the feedback, i.e., they incorporate new information into the previous estimate, resulting in an improved state estimate [11].

Forecasting equations can alternatively be defined as equations that update in real-time, while correction equations incorporate new information. A forecast-correction method for various issues can be used to characterize the final estimation procedure. To correct the projection using the new measurement and forecast the new state and its uncertainty, the Kalman filter employs a projection correction mechanism. This cycle is shown in Figure 2.

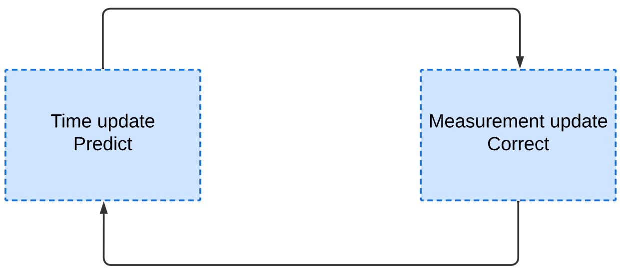

Figure 2: Kalman Filter Cycle

The first step consists of generating a forecast of the future state in time, taking into account all the information available at that time, and in the second step, an improved prediction of the state is generated so that the error is statistically minimized. The specific equations for forecasting and status correction are given below.

$$\hat{x}^{-} = A\hat{x}_{k-1} + Bu_k \tag{5}$$

$$P_x^{-} = AP_{k-1}A^T + Q \tag{6}$$

The equations in Table 2 forecast the state estimates and the forward covariance from k-1 to k. Matrix A relates the state at the previous time k-1 to the form at the current time k; this matrix could change for different points in time (k). Q represents the covariance of the random disturbance of the process trying to estimate the state.

$$K_k = P_k^{-} H^T (HP_k^{-} H^T + R)^{-}1 \tag{7}$$

$$\hat{x}_k = \hat{x}_k^{-} + K_k(Z_k - H\hat{x}_k^{-}) \tag{8}$$

$$P_k = (I - K_k H)P_k^{-} \tag{9}$$

The first task during the correction of the state projection is calculating the Kalman gain, K_k, using equation (7). This weighting factor or growth is selected to minimize the error covariance of the new state estimate. The next step is to measure the process to obtain Z_k and then generate a new state estimate incorporating the new observation as in equation (8). The final step is to get a new assessment of the error covariance using equation (9).

After each pair of updates, both time and measurement, the process is repeated, taking the new state and error covariance estimates as a starting point. This recursive nature is one of the striking features of the Kalman filter [4, 7]. Figure 3 gives a complete picture of the operation of the filter, combining Figure 2 with equations (5), (6), (7), (8) and (9). The KF uses the least squares method to recursively generate an estimator of the state at time k, which is linear, unbiased, and of minimum variance. The filter is in line with the Gauss-Markov theorem, giving the KF enormous power to solve a wide range of problems in statistical inference.

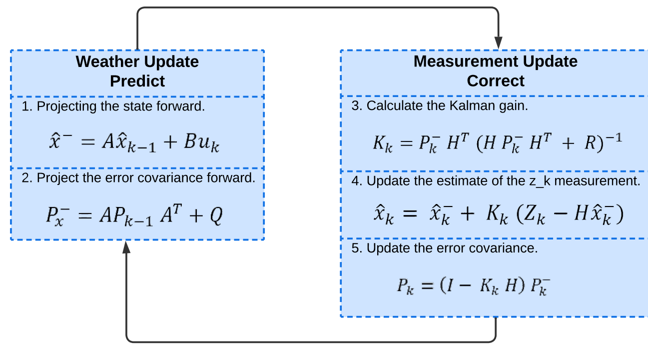

Figure 3: Kalman Filter Overview

It avoids the influence of possible structural changes in the estimation. Selective estimation starts with an initial sample and updates the estimates by successively incorporating new observations until all the information is covered. This means that the most recent assessment of the factors is influenced by the long history of the series, which, in the case of structural changes, could distort it. Sequential estimations can correct this bias, although doing so will increase the standard error. As a result, the KF employs the complete series history, much like recursive methods do. It does, however, have the benefit of removing the chance of estimation bias in structural modifications by aiming to estimate a stochastic route for the coefficients instead of a deterministic one.

The filter stands out because it can predict the state of a model in the past, present, and future, even when the modeled system's specific characteristics are unclear. One of the main distinguishing features of the Kalman approach is the dynamic modeling of a system. A linear transition from one period to the next characterizes linear dynamic models, which include most of the models frequently used in time series analysis.

One of the filter's drawbacks is that to initiate the recursive algorithm, initial conditions of the mean and variance of the state vector must be met as starting conditions. There is still disagreement about what these starting circumstances should be. For example, in a Bayesian approach, this filter requires a priori values of the initial coefficients and their respective variances to be specified. One way may be to obtain this information by estimating a model similar to the desired one but with fixed coefficients for a sample subperiod. On the other hand, it is necessary to specify the variances for which minimal and proportional conflicts are suggested about those obtained for the initial coefficients [12].

The development of the Kalman filter, as found in the original paper, assumes a broad knowledge of probability theory, specifically with the issue of Gaussian conditionality in random variables, which may cause a limitation for its study and application. When developed for autoregressive models, the results are conditioned to the past information of the variable in question. In this sense, time series forecasting represents the strength or inertia currently in the system and is efficient only in the short term.

2.3. Fuzzy Logic

Fuzzy logic (FL) is an extension of Boolean logic by Lotfi Zadeh in 1965 based on the mathematical theory of fuzzy sets, which is a generalization of classical set theory [13], has been developed basically in different disciplinary practices, especially in those related to industrial process control, the computer sector and numerous applications in economics [14, 15].

Fuzzy logic can be defined as:

- Mathematics that generalizes two-valued (0,1) logic for reasoning under uncertainty.
- Theories and technologies that employ fuzzy sets, which are set with boundaries based on a membership degree [16].

The first goal of fuzzy logic is to alleviate difficulties in developing and analyzing complex systems involving conventional mathematical tools [17]. It is also motivated by the assumption that human reasoning does not always have well-established boundaries. Fuzzy logic can be used to model and control complex and nonlinear systems or systems that need to be better defined for conventional modeling and control techniques. Fuzzy logic is a technology for developing intelligent control and information systems, as it offers a practical way to design nonlinear control systems. It achieves nonlinearity through a linear approximation of the system elements. The basic building block for fuzzy control systems is the if-then rule set, which performs a functional mapping [18].

Fuzzy logic techniques are based on four basic concepts:

- Fuzzy sets are those having smooth boundaries. Ambiguity.
- Linguistic variables are those whose values fit into a fuzzy set and can be qualitative and quantitative.
- Distribution of Possibilities: Assigning a fuzzy set imposes a linguistic variable's value limitation.
- A fuzzy if-then rule is a knowledge representation scheme for a functional mapping or a logical formula that generalizes an implication into two logical values.

Fuzzy Sets allow the elimination of fixed and exact constraints by using the membership of a set through its degree of membership. A group's membership degree is expressed by a number between zero and one, where zero means altogether outside the scene, one means entirely in the set, and a number between zero and one means partially inside the group. In this way, a gradual and smooth transition can be described from outside the location to inside the set. So, a fuzzy set is defined by a function that maps objects in a domain concerning their membership value in the group. It is important to remember that a fuzzy set is always expressed in a context, even if the context is not explicit [19].

Linguistic variables allow their value to be described qualitatively (linguistic term) and quantitatively (corresponding to a membership value). The linguistic term is used to express concepts and knowledge in human communication, while the membership function is helpful for input data processing. A linguistic variable is a composition of a symbolic and numeric variable. For situations with a very definite boundary between the possible and the impossible, fuzzy logic offers an alternative, in which it generalizes the distinction between the possible and the impossible through a degree of possibility.

Fuzzy if-then rules have been applied to many disciplines, such as control systems, decision-making, pattern recognition, and system modeling. Conceptually, these rules generalize a logical inference called Modus Ponens, in which the inferred conclusions are modified by a degree of membership in which the antecedent is satisfied. Mathematically, it can be seen as an interpolation scheme because it allows the fusion of multiple fuzzy rules when all their conditions are comfortable to a certain degree [16].

This theory is based on sets of fuzzy or fuzzy numbers, which denote, in essence, groups of elements belonging with varying intensities or degrees to a specific category. It allows multiple levels between the extreme values of each interval, even with the opportunity to establish references of resemblance between the limits and their internal nuances [15].

The above contrasts with the ideal world posed by Classical Logic (CL), which is based on the membership or not of the elements to each category. Consequently, LD has a more remarkable resemblance to the reality of social phenomena, where expressions are used whose boundaries are not clearly defined, as in the case of the terms familiar, quickly, approximate, old, novice, warm, experienced, fleeting, firm, submissive, authoritarian, etc., making it possible to classify an object or phenomenon into several conceptual categories at the same time, depending on the scale used by the person making the judgment.

In this representation (Figure 4), the concept that is qualified in a fuzzy way is the linguistic variable, while the different values it takes or is associated with constitute the linguistic values. In addition, each linguistic value is, in turn, another fuzzy set, and the range of values that the linguistic variable can take is known as the Discourse Universe, Universal Set, or simply Domain (U).

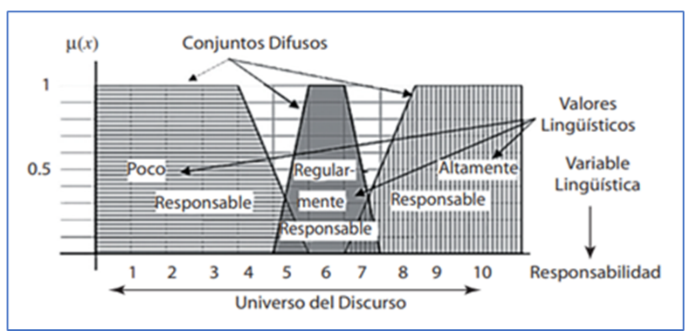

Figure 4: Belonging Functions

The Belonging Function (μ) assigns to each element of U a degree of membership or belonging to the fuzzy set, which is always in the interval [0, 1]; if it takes the value one (1), it means that it fully complies with the condition or characteristic of the fuzzy set, while a value of zero (0) would be equivalent to saying that the element does not meet that condition.

The difference between FL and CL is that the former can establish degrees of membership to an element of the set, which implies the validity of partial membership (valuations between 0 and 1). At the same time, in the latter, this is impossible since CL only allows total membership (value 1) or exclusion (value 0) to each category [20].

3. Results

3.1. Diffuse Kalman Filter

Based on the discrete Kalman filter presented in the previous subsection, it is used to design a fuzzy control to manage the covariance matrix as it directly affects the filter performance. It has been determined that the main equations that model a Kalman Filter are given in table 1:

Table 1: Main Equations that model the Kalman Filter

Item	Equation	Description
1	$P_X^- = AP_{K-1}A^T + Q$	A priori value of the covariance of the estimated error.
2	$K_K = P_K^- H^T (HP_K^- H^T + R)^{-1}$	Calculation of the correction gain.
3	$P_K = (I - K_K H)P_K^-$	A posteriori value of the covariance of the estimated error.

It can be seen in Figure 5 that between the variables P_K^- (a priori value of the estimated error covariance) and P_K (a posteriori value of the estimated error covariance), there is a difference that serves as input to the fuzzy control, as proposed in [21]. The other input value to the system is P_K, which serves to know the covariance of the error of the state vector after correction and to monitor how it approaches zero.

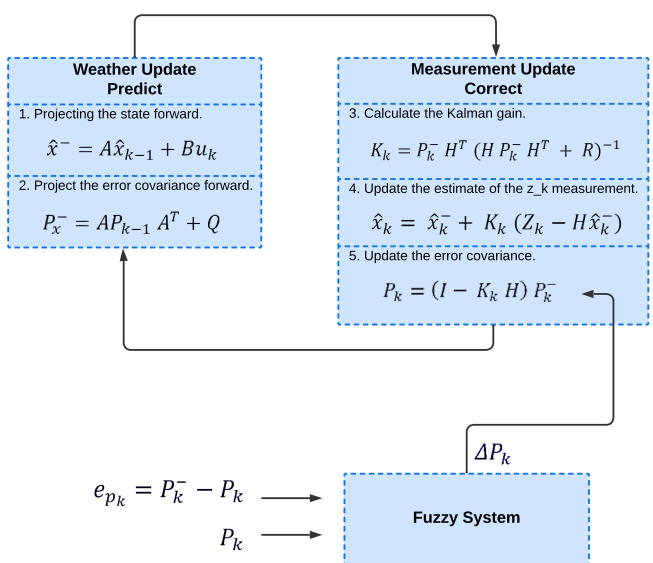

Figure 5: Operation of the Kalman Filter with Fuzzy control

The output of the fuzzy system is the value of the decrement/increment that is made to that same value of P_K. This forces the system to approach a covariance error equal to zero faster, depending on the covariance's position and the error between the two covariance values (a priori and a posteriori). A general diagram of the filter operation is shown in Figure 5.

The fuzzy system rules were represented (Table 2) because the input P_k has the values of [Z, SP, LP] (Zero, Small Positive, and Large Positive), the input e_{P_k} has the values of [LN, SN, Z] (Large Negative, Small Negative and Zero), and the output ΔP_k can take the values of [LN, SN, Z] (Large Negative, Small Negative and Zero).

Table 2: Values in the fuzzy rules.

Item	Inputs	Values	Description
1	P_k	[Z, SP, LP]	(Zero, Small Positive and Large Positive)
2	e_{P_k}	[LN, SN, Z]	(Large Negative, Small Negative and Zero)
3	ΔP_k	[LN, SN, Z]	(Large Negative, Small Negative and Zero)

Figure 6 shows the rule matrix for the Diffuse Kalman Filter and figure 7 shows the diffuse fuzzy control with two inputs P_k, e_{P_k} and an output ΔP_k.

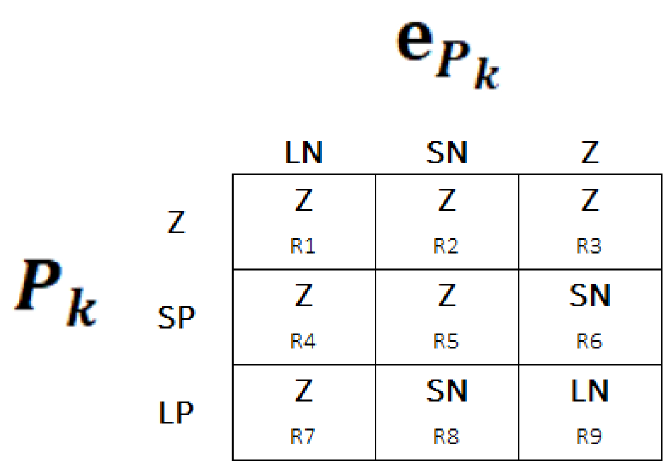

Figure 6: Rule matrix for the Diffuse Kalman filter

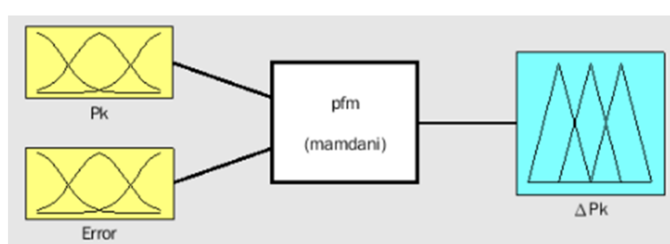

Figure 7: Diffuse Control

The following table 3 describes the rules to be used in fuzzy control:

Table 3: Values in the fuzzy rules.

Item	Rules
1	If (P_K is Z) and (e_{P_k} is LN) then (ΔP_k is Z)
2	If (P_K is Z) and (e_{P_k} is SN) then (ΔP_k is Z)
3	If (P_K is Z) and (e_{P_k} is Z) then (ΔP_k is Z)
4	If (P_K is SP) and (e_{P_k} is LN) then (ΔP_k is Z)
5	If (P_K is SP) and (e_{P_k} is SN) then (ΔP_k is Z)
6	If (P_K is SP) and (e_{P_k} is Z) then (ΔP_k is SN)
7	If (P_K is LP) and (e_{P_k} is LN) then (ΔP_k is Z)
8	If (P_K is LP) and (e_{P_k} is SN) then (ΔP_k is SN)
9	If (P_K is LP) and (e_{P_k} is Z) then (ΔP_k is LN)

The rules are interpreted as follows: for the first rule, if P_k is close to zero and the value of e_{P_k} is large negative, meaning that the difference between $P_k^- - P_k$ is large, it can be deduced that P_k is a value close to zero and is approaching quickly, so no value needs to be added or subtracted.

For the second rule if P_k is close to zero and the value of e_{P_k} is small negative, that is, the difference between $P_k^- - P_k$ is small, it can be deduced that P_k is a value close to zero and is fast approaching, so no value needs to be added or subtracted.

For the third rule, if P_k is close to zero and the value of e_{P_k} is zero, meaning that the difference between $P_k^- - P_k$ is zero, it can be deduced that P_k is a heat close to zero and is approaching fast, so no value needs to be added or subtracted.

For the fourth rule, if P_k is a small positive value and the value of e_{P_k} is a large negative, that means the difference between $P_k^- - P_k$ is large, it can be deduced that P_k is a value close to zero, so no value needs to be added or subtracted from it.

For the fifth rule, if P_k is a small positive value and the value of e_{P_k} is a small negative, meaning that the difference between $P_k^- - P_k$ is small, it can be deduced that P_k is a value close to zero, so no value needs to be added or subtracted.

For the sixth rule, if P_k is a small positive value and the value of e_{P_k} is zero, meaning that the difference between $P_k^- - P_k$ is zero, it can be deduced that P_k is a value that is not close to zero, so a small value needs to be subtracted from P_k to make it try to approach zero faster.

For the seventh rule, if P_k is a large positive value and the value of e_{P_k} is a large negative, meaning that the difference between $P_k^- - P_k$ is large, it can be deduced that P_k is a value close to zero, so no value needs to be added or subtracted from it.

For the eighth rule, if P_k is a large positive value and the value of e_{P_k} is a small negative, meaning that the difference between $P_k^- - P_k$ is small, it can be deduced that P_k is a value that is not close to zero, so no small value needs to be subtracted from it.

For the ninth rule, if P_k is a large positive value and the value of e_{P_k} is zero, this means that the difference between $P_k^- - P_k$ is large, it can be deduced that P_k does not try to approach zero, so it is not necessary to subtract a large value from P_k to make it try to approach zero faster.

Figures 8, 9, and 10 show the membership functions used by the fuzzy control. Based on the above rules, it was decided to add a fuzzy part to the existing simulation, so the MATLAB Fuzzy Logic toolbox was used, with the following features:

- Fuzification : Singleton uncertainty.
- T-Norm : Minimal.
- Implication : Mamdani
- Defuzzification : Centroid

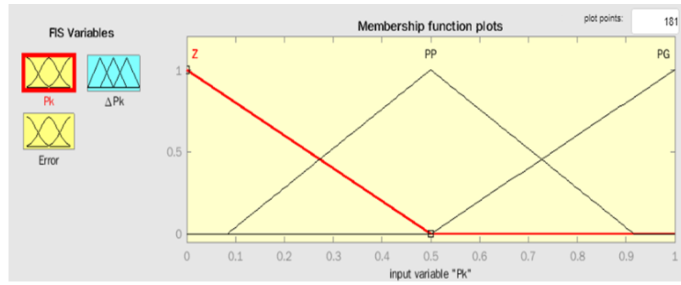

Figure 8: P_K Membership Function

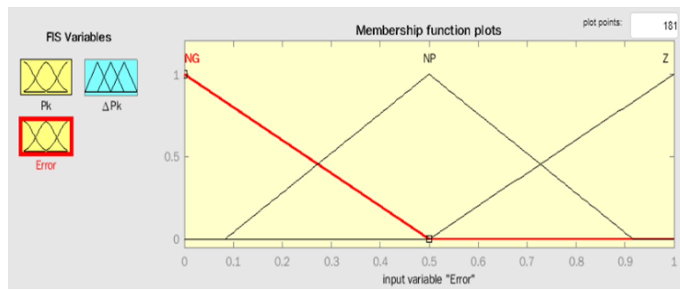

Figure 9: e_{P_K} Membership Function

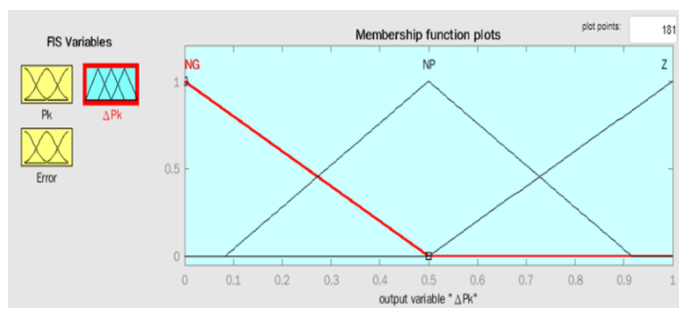

Figure 10: ΔP_K Membership Function

3.2. Simulation

This section analyzes, compares, and interprets the simulations of the KF and the DKF with the characteristic values of a GPS sensor.

3.2.1. Latitude

Figure 11 shows the comparison between the two simulations, showing that the error in the latitude processing of the DKF approaches a value closer to zero than the conventional KF.

Figure 12 shows the comparison of the prediction of the Kalman Filter versus the Diffuse Kalman filter on the latitude signal concerning time, and it can be seen that the FKF prediction fits better concerning the original signal with noise.

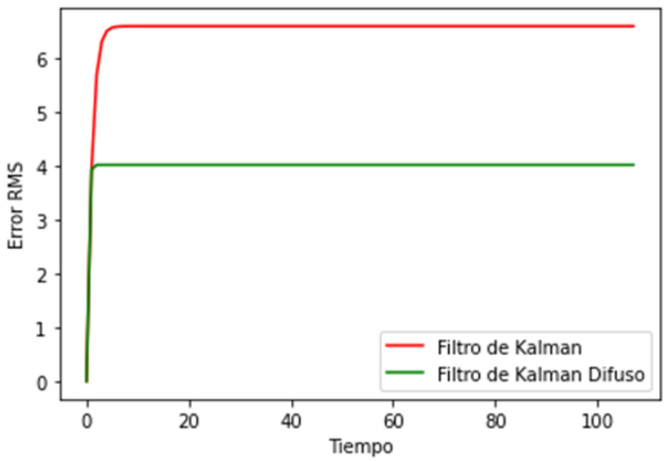

Figure 11: Comparison of Latitude Error (KF vs FKF)

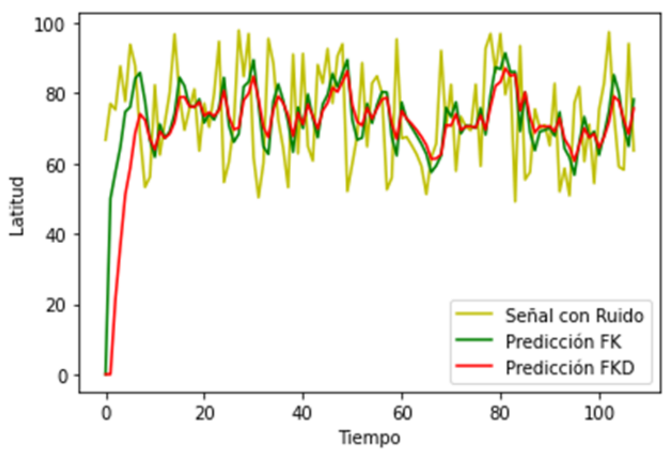

Figure 12: Comparison of Latitude Prediction (KF vs FKF)

Figure 13 shows the comparison of the Kalman Filter versus the Diffuse Kalman filter update on the latitude signal concerning time, and it can be seen that the FKF update fits better concerning the original signal with noise.

3.2.2. Longitude

Figure 14 shows the comparison between the two simulations of the conventional Kalman filter and the Diffuse Kalman filter, where it is noted that the longitude error in the Diffuse Kalman filter approaches a value closer to zero than the conventional Kalman filter.

Figure 15 shows the comparison of the prediction of the Kalman Filter versus the Diffuse Kalman filter on the longitude signal concerning time, and it can be seen that the FKF prediction fits better concerning the original signal with noise.

Figure 16 shows the comparison of the Kalman Filter versus Diffuse Kalman filter update on the longitude signal concerning time and it can be seen that the FKF update fits better concerning the original signal with noise.

3.2.3. Altitude

Figure 17 shows the comparison between the two simulations of the conventional Kalman filter and the Diffuse Kalman filter, where it is noted that the altitude error in the Diffuse Kalman filter approaches a value closer to zero than the conventional Kalman filter.

Figure 18 shows the comparison of the prediction of the Kalman Filter versus the Diffuse Kalman filter on the altitude signal concerning time and it can be seen that the FKF prediction fits better concerning the original signal with noise.

Figure 19 shows the comparison of the Kalman Filter versus Diffuse Kalman filter update on the altitude signal concerning time and it can be seen that the FKF update fits better concerning the original signal with noise.

4. Discussion

KF is a widely used filtering technique for predicting the state of a dynamic system considering noisy measurements. However, traditional KF is designed for linear and Gaussian systems, which limits its applicability to complex and nonlinear systems. The DKF is an extension of the KF that allows estimation in nonlinear and non-Gaussian systems by combining the Kalman Filter with fuzzy logic [22].

Fuzzy logic helps to represent uncertainty and imprecision in complex systems. DKF uses fuzzy sets instead of probability distributions to represent system states and measurements. This helps to handle noisy measurements better and capture uncertainty in nonlinear systems.

The advantage of the DKF is that it can handle complex, nonlinear systems, making it applicable in a wide range of areas, such as robotics, navigation, economics, and biomedicine. In addition, DKF provides more accurate and robust estimation than other nonlinear methods, such as particle filters. However, DKF also has some limitations. The main one is computational complexity, as the computation of fuzzy logic and updating estimates may require more computational resources than the traditional Kalman filter. In addition, proper selection of the fuzzy sets and DKF parameter settings can be challenging and require specialized knowledge.

DKF can handle noisy measurements and non-Gaussian error distributions by combining fuzzy logic with the Kalman filter. This provides a more accurate and robust estimation of the system state, which is crucial in applications where accuracy is essential, such as real-time navigation or complex process monitoring.

DKF can also handle smooth transitions between various models or regimes, making it more adaptable and flexible. This is particularly crucial in dynamic environments where systems face disturbances, condition changes, or unforeseen events.

Using the KF and DKF to treat the GPS signal proved an excellent option for reducing sensor error. However, there are much more efficient techniques, such as the particle filter, that effectively treat non-Gaussian noise and nonlinear models, especially in applications where the computational cost is cheap and the sampling rate is moderate, so this line of research can obtain essential contributions. Still, it must be addressed in the present work since it is not part of the stated objectives.

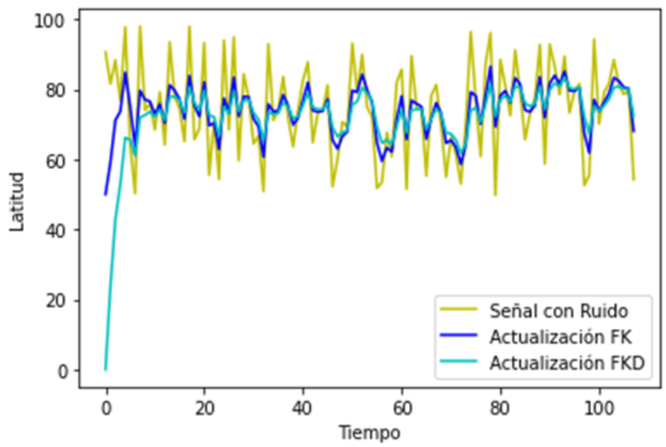

Figure 13: Latitude Updating Comparison (KF vs FKF)

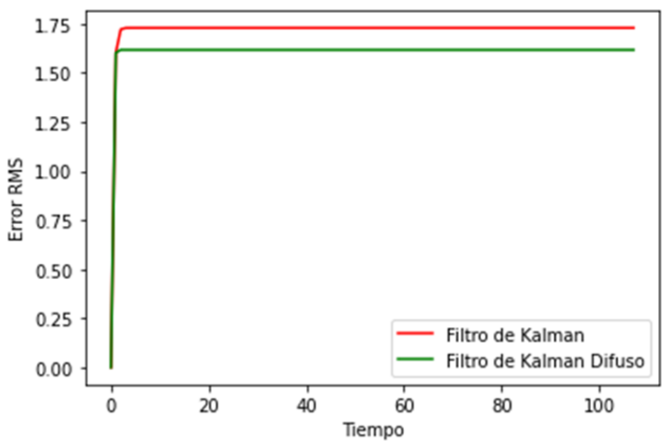

Figure 14: Comparison of longitude Error (KF vs FKF)

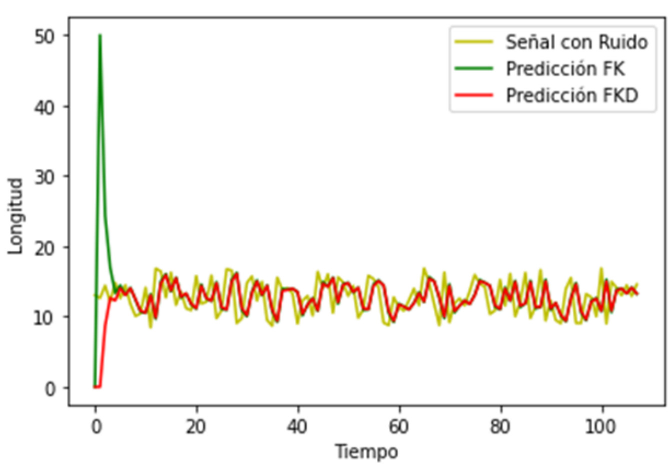

Figure 15: Comparison of longitude Prediction (KF vs FKF)

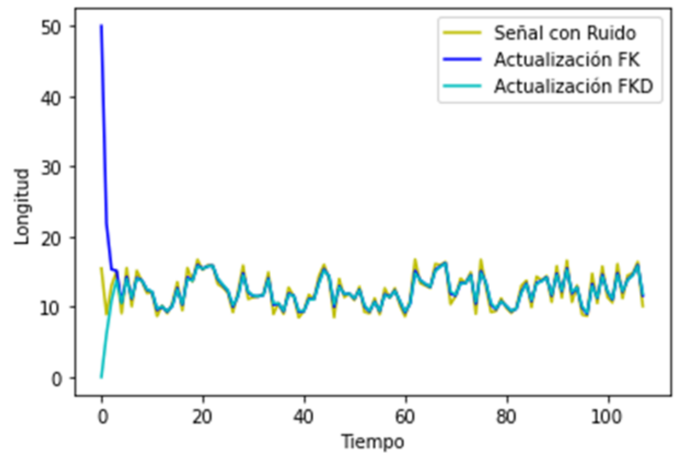

Figure 16: Comparison of longitude Updating (KF vs FKF)

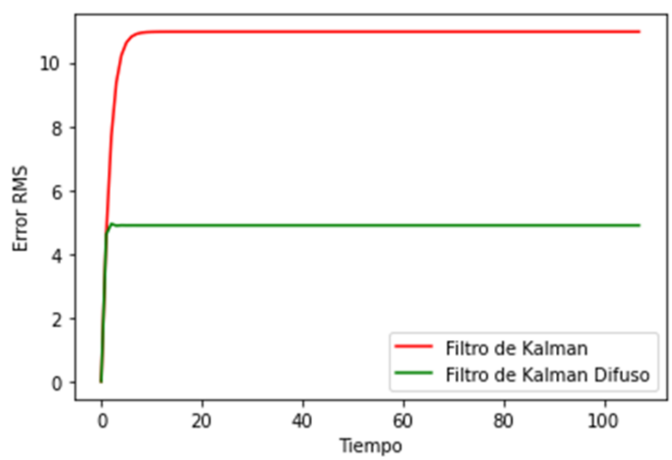

Figure 17: Comparison of Altitude Error (KF vs FKF)

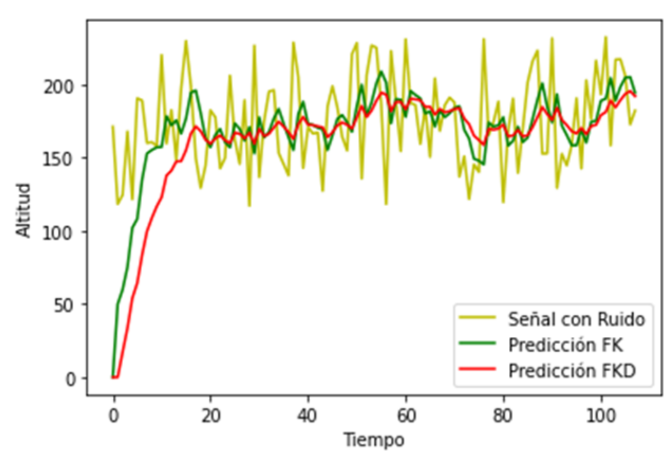

Figure 18: Comparison of Altitude Prediction (KF vs FKF)

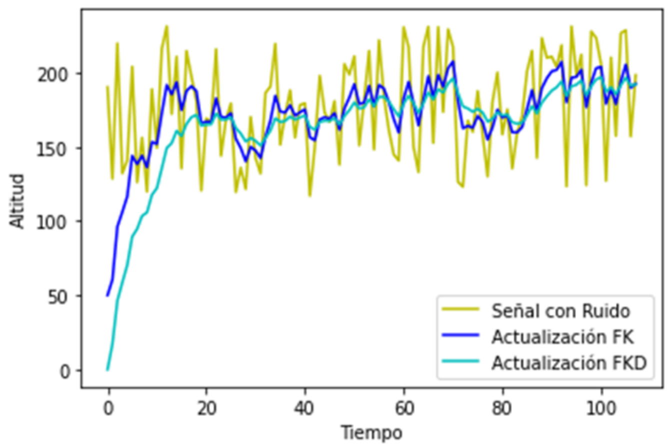

Figure 19: Comparison of Altitude Updating (KF vs FKF)

5. Conclusions

This work proposes using the fuzzy Kalman filter with a GPS sensor to maximize the accuracy in the prediction of position and velocity in an autonomous vehicle. This work was divided into 3 essential parts: the Kalman filter, the fuzzy control, and the simulation of the signal from a GPS sensor, taking into account that in this last part, a comparison is made in the behavior between the discrete Kalman filter and the Fuzzy Kalman filtert.

In general, high-reliability values are achieved with position estimation in GPS measurements. The variations depend very much on how the position measurements are taken to model the noise to which it is exposed. One of the advantages of the Kalman filter is that it avoids the purchase of costly and accurate sensors since if sensors with more accurate clocks and more advanced position measurement techniques were considered, the costs would rise considerably.

Finally, the benefit of using techniques related to Fuzzy Logic in the design of fuzzy control is compelling, as it reduces the position estimation error. The fuzzy Kalman filter is worth studying because of its ability to handle complex and nonlinear systems, improve the robustness and accuracy of estimates, incorporate expert information, and adjust to model changes. Because of these qualities, the fuzzy Kalman filter is a valuable tool in many estimation problems. A comparison and analysis of the Fuzzy Kalman Filter, the Extended Kalman Filter, and the Particle Filter is suggested for future work.

References

[1] R. E. Kalman, "A New Approach to Linear Filtering and Prediction Problems." ASME. J. Basic Eng., 82(1): 35–45, 1960, doi:10.1115/1.3662552.

[2] E. A. Wan, R. van der Merwe, "The unscented Kalman filter for nonlinear estimation," Proceedings of the IEEE 2000 Adaptive Systems for Signal Processing, Communications, and Control Symposium (Cat. No.00EX373), 153–158, 2000.

[3] D.-J. Jwo, S.-Y. Lai, "Navigation Integration Using the Fuzzy Strong Tracking Unscented Kalman Filter," The Journal of Navigation, **62**(2), 303–322, 2009, copyright - Copyright © The Royal Institute of Navigation 2009; Última actualización - 2023-11-28.

[4] C. K. Chui, G. Chen, "Kalman Filtering with Real-Time Applications," Springer Berlin, Heidelberg, XV, 191, 2009, doi:10.1007/978-3-662-02508-6.

[5] J. T. Jalles, "Structural Time Series Models and the Kalman Filter: A Concise Review," SSRN, **541**, 153–158, 2009, doi:10.2139/ssrn.1496864.

[6] G. Bishop, G. Welch, et al., "An introduction to the kalman filter," Proc of SIGGRAPH, Course, **8**(27599-23175), 41, 2001.

[7] R. J. Meinhold, N. . D. Singpurwalla, "Understanding the kalman filter," Am. Stat., **vol. 37**(No. 2), 123–127, 1983.

[8] H. W. Sorenson, "Least-squares estimation: from Gauss to Kalman," IEEE Spectrum, **7**(7), 63–68, 1970, doi:10.1109/MSPEC.1970.5213471.

[9] B. Yildirim, Sigmarho Kalman Filter Implementation and Analysis, Ph.D. thesis, California State University, 2020, copyright - Database copyright ProQuest LLC; ProQuest does not claim copyright in the individual underlying works; Última actualización - 2023-06-21.

[10] A. Khalid, R. K. Syed Abdul, N. U. Ain, M. Awais, A. S. Majid, J. E. M. Carreño, J. C. Vasquez, J. M. Guerrero, B. Khan, "Comparison of Kalman Filters for State Estimation Based on Computational Complexity of Li-Ion Cells," Energies, **16**(6), 2710, 2023.

[11] M. Grewal, A. Andrews, "Kalman filtering: theory and practice using MATLAB," New York: John Wiley and Sons, **14**, 2001, doi:10.1002/9780470377819.

[12] R. L. Thomas Doan, C. Sims, "Forecasting and conditional projection using realistic prior distributions," Econometric Reviews, **3**(1), 1–100, 1984, doi:10.1080/07474938408800053.

[13] J. A. Goguen, "L. A. Zadeh. Fuzzy sets. Information and control, vol. 8 (1965), pp. 338–353. - L. A. Zadeh. Similarity relations and fuzzy orderings. Information sciences, vol. 3 (1971), pp. 177–200." The Journal of Symbolic Logic, **38**(4), 656–657, 1973, doi:10.2307/2272014.

[14] T. J. Ross, "Fuzzy Logic with Engineering Applications," Wiley, **3**, 1–100, 2010, doi:10.1002/9781119994374.

[15] I. P. B. Leon, "Logica difusa para principiantes. Teoria y practica," Publicaciones UCAB, **3**, 2007.

[16] J. Yen, R. Langari, "Fuzzy logic: Intelligence, control, and information," Prentice Hall, Upper Saddle River NJ, 1999.

[17] C. Ni, X. Ma, "An integrated long-short term memory algorithm for predicting polar westerlies wave height," Ocean Engineering, **215**, 107715, 2020, doi:https://doi.org/10.1016/j.oceaneng.2020.107715.

[18] C. Lu, "Concrete materials compressive strength using soft computing techniques," Multiscale and Multidisciplinary Modeling, Experiments and Design, 2023, doi:10.1007/s41939-023-00276-4.

[19] R. Srivastava, M. P. S. Bhatia, "Quantifying modified opinion strength: A fuzzy inference system for Sentiment Analysis," in 2013 International Conference on Advances in Computing, Communications and Informatics (ICACCI), 1512–1519, 2013, doi:10.1109/ICACCI.2013.6637404.

[20] M. G. Voskoglou, "Fuzzy Logic as a Tool for Assessing Students' Knowledge and Skills," Education Sciences, **3**(2), 208–221, 2013, doi:10.3390/educsci3020208.

[21] R. Woo, E.-J. Yang, D.-W. Seo, "A Fuzzy-Innovation-Based Adaptive Kalman Filter for Enhanced Vehicle Positioning in Dense Urban Environments," Sensors, **19**(5), 2019, doi:10.3390/s19051142.

[22] B. Parrell, V. Ramanarayanan, S. Nagarajan, J. Houde, "The FACTS model of speech motor control: fusing state estimation and task-based control," bioRxiv, 2019, doi:10.1101/543728.