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Model Reduction H_∞ Finite Frequency of Takagi-Sugeno Fuzzy Systems

Rim Mrani Alaoui[1], Abderrahim El-Amrani[2,*], Ismail Boumhidi[2]

[1] *Higher Institute of Engineering and Business, ISGA, Fez, 30000, Morroco*

[2] *LISAC Labs, Sidi Mohamed Ben Abdellah University, Fez, 30000, Morocco*

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ABSTRACT

The daily treats model reduction finite frequency (FFMR) design for Takagi Sugeno (T S) systems. This work is to FFMR design in such a way whether augmented model is steady get a reduced H_∞ index in FF areas with noise is established as a prerequisite. To highlight the importance of suggested process, a practical application has been made.

1 Introduction

Fuzzy models T-S [1] call for wide observation of various practical industrial applications, mainly as the recognized T-S samples actually approximate nonlinear shapes. The essential characteristic on the sample TS is its general estimation of a nonlinear function. There are large number of results of literature that treat the globally difficulties utilizing the TS fuzzy samples, see [2]–[7].

The our existing sources on model reduction problem and disturbances are based on the whole full frequency (EF) area, which will give several types of model reduction design [8]–[13]. However, most practical industrial applications work in a FF domain. So far, a few applications have been made [14]–[19]. Thus, for this we will present new approaches to solve these problems.

The primary goal of our work is to define a fuzzy model reduction of discrete Model over FF ranges such a way that the augmented model is steady get a reduced H_∞ index in FF areas with disturbance is established as a prerequisite. we have also presented an example of simulation in order to exemplify the efficiency of the suggested method.

Notations :

- "T" : Matrix transposition
- " $*$ " : Matrix symmetry
- $M > 0$: matrix M is positive
- **sym**$[\mathcal{D}]$: $\mathcal{D} + \mathcal{D}^*$
- **He**$[\mathcal{D}]$: $\frac{\mathcal{D}+\mathcal{D}^*}{2}$

2 Problem statement

2.1 System formulation

Envisage the nonlinear model presented by :

Rule x: IF $\zeta_1(\mu)$ is T_1^i,... $\zeta_n(\mu)$ is $\mathbf{T}_n^i$ THEN

$$\begin{aligned} \mathbf{z}_{\mu+1} &= \mathbf{M}_x\mathbf{z}(\mu) + \mathbf{N}_x\mathbf{d}(\mu) \\ \mathbf{w}(\mu) &= \mathbf{J}_x\mathbf{z}(\mu) \end{aligned} \tag{1}$$

(2)

with $\mathbf{z}(\mu)$ is the input; $\mathbf{w}(\mu)$ is deliberate output; $\mathbf{M}_x$, $\mathbf{N}_x$ and $\mathbf{J}_x$ are system parametres; $\zeta_1(\mu)$,..., $\zeta_n(\mu)$ the premise variables. $\mathbf{d}(\mu)$ is a known disturbance signal located in a following FF areas

$$\varrho \;=\; \left\{ \;\tilde{\varrho} \in \mathbb{R}| \quad \tilde{\varrho}_1 \le \tilde{\varrho} \le \tilde{\varrho}_2; \qquad \tilde{\varrho}_1, \tilde{\varrho}_2 \in [-\pi, +\pi], \right. \tag{3}$$

We describe the nonlinear system (1) employ singleton fuzzifer, center-average and inference product by the following relation :

$$\begin{aligned} \mathbf{z}_{\mu+1} &= \mathbf{M}(\mathbf{l})x(\mu) + \mathbf{N}(\mathbf{l})\mathbf{d}(\mu) \\ \mathbf{w}(\mu) &= \mathbf{J}(\mathbf{l})\mathbf{z}(\mu) \end{aligned} \tag{4}$$

[*] A. EL-AMRANI & Email : abderrahim.elamrani@usmba.ac.ma

where

$$
\begin{aligned}
\mathbf{M}(\mathbf{l}) &= \sum_{x=1}^{q} \mathbf{l}_x \mathbf{M}_x; \quad \mathbf{N}(\mathbf{l}) = \sum_{x=1}^{q} \mathbf{l}_x \mathbf{N}_x; \\
\mathbf{J}(\mathbf{l}) &= \sum_{x=1}^{q} \mathbf{l}_x \mathbf{J}_x
\end{aligned} \tag{5}
$$

In this work, a fuzzy MR function is represented by :

Rule x: if $\zeta_1(\mu)$ is $\mathbf{T}_1^x$... $\zeta_n(\mu)$ is $\mathbf{T}_n^x$ then

$$
\begin{aligned}
\hat{\mathbf{z}}(\mu+1) &= \hat{\mathbf{M}}_x \hat{\mathbf{z}}(\mu) + \hat{\mathbf{N}}_x \mathbf{d}(\mu) \\
\hat{\mathbf{w}}(\mu) &= \hat{\mathbf{J}}_x \hat{\mathbf{z}}(\mu)
\end{aligned} \tag{6}
$$

with $\hat{\mathbf{z}}(\mu)$ is state MR vector; $\hat{\mathbf{w}}(\mu)$ is the output MR function, $\hat{\mathbf{M}}_x$, $\hat{\mathbf{N}}_x$ and $\hat{\mathbf{J}}_x$ are parameters should exist defined.

We get defuzzified for system (6) as following :

$$
\begin{aligned}
\hat{\mathbf{z}}(\mu+1) &= \hat{\mathbf{M}}(\hat{\mathbf{l}}) \hat{\mathbf{z}}(\mu) + \hat{B}(\hat{\mathbf{l}}) \mathbf{d}(\mu) \\
\hat{\mathbf{w}}(\mu) &= \hat{\mathbf{J}}(\hat{\mathbf{l}}) \hat{\mathbf{z}}(k)
\end{aligned} \tag{7}
$$

with

$$
\begin{aligned}
\hat{\mathbf{M}}(\hat{\mathbf{l}}) &= \sum_{x=1}^{q} \hat{\mathbf{l}}_x \hat{\mathbf{M}}_x, \quad \hat{B}(\hat{\mathbf{l}}) = \sum_{x=1}^{q} \hat{\mathbf{l}}_x \hat{\mathbf{N}}_x, \\
\hat{\mathbf{J}}(\hat{\mathbf{l}}) &= \sum_{x=1}^{q} \hat{\mathbf{l}}_x \hat{\mathbf{J}}_x
\end{aligned} \tag{8}
$$

Consider $\nabla(\mu) = \begin{bmatrix} \mathbf{z}(\mu) \\ \hat{\mathbf{z}}(\mu) \end{bmatrix}$, $\mathbf{g}(\mu) = \mathbf{w}(\mu) - \hat{\mathbf{w}}(\mu)$. Then, the error system is as follows:

$$
\begin{aligned}
\nabla(\mu+1) &= \bar{\mathbf{M}}(\mathbf{l}, \hat{\mathbf{l}}) \nabla(\mu) + \bar{\mathbf{N}}(\mathbf{l}, \hat{\mathbf{l}}) \mathbf{d}(\mu) \\
\mathbf{g}(\mu) &= \bar{\mathbf{J}}(\mathbf{l}, \hat{\mathbf{l}}) \nabla(\mu)
\end{aligned} \tag{9}
$$

where

$$
\begin{aligned}
\bar{\mathbf{M}}(\mathbf{l}, \hat{\mathbf{l}}) &= \begin{bmatrix} \mathbf{M}(\mathbf{l}) & 0 \\ 0 & \hat{\mathbf{M}}(\hat{\mathbf{l}}) \end{bmatrix}; \quad \bar{\mathbf{N}}(\mathbf{l}, \hat{\mathbf{l}}) = \begin{bmatrix} \mathbf{N}(\mathbf{l}) \\ \hat{\mathbf{N}}(\hat{\mathbf{l}}) \end{bmatrix}; \\
\bar{\mathbf{J}}(\mathbf{l}, \hat{\mathbf{l}}) &= \begin{bmatrix} \mathbf{J}(\mathbf{l}) & -\hat{\mathbf{J}}(\hat{\mathbf{l}}) \end{bmatrix}
\end{aligned} \tag{10}
$$

Definition 2.1 *[20] Consider $\beta > 0$, if the following inequality is verified:*

$$
\sum_{\mu=0}^{\infty} \boldsymbol{g}^T(\mu) \boldsymbol{g}(\mu) \le \beta^2 \sum_{\mu=0}^{\infty} \boldsymbol{d}^T(\mu) \boldsymbol{d}(\mu) \tag{11}
$$

$$
\tag{12}
$$

From Parseval function [21], we get :

$$
\int_{-\pi}^{\pi} \boldsymbol{g}^T(\varrho) \boldsymbol{g}(\varrho) d\varrho \le \beta^2 \int_{-\pi}^{\pi} \boldsymbol{d}^T(\varrho) \boldsymbol{d}(\varrho) d\varrho \tag{13}
$$

$$
\tag{14}
$$

We express the question of this work by : we design an appropriate fuzzy MR system (7) such that a error model is well-posed, stable satises the FF index:

$$
\int_{\varrho \in \varrho} \mathbf{g}^T(\varrho) \mathbf{g}(\varrho) d\varrho \le \beta^2 \int_{\varrho \in \varrho} \mathbf{d}^T(\varrho) \mathbf{d}(\varrho) d\varrho \tag{15}
$$

2.2 Preliminaries

Lemma 2.2 *[22] Let $\Phi \in \mathbb{R}^m$, $\mathcal{L} \in \mathbb{R}^{m \times m}$ and $\mathcal{S} \in \mathbb{R}^{p \times m}$. Then, The following equations are the same:*

- $\Phi^* \mathcal{L} \Phi < 0, \forall \Phi \neq 0 : \mathcal{S}\Phi = 0$
- $\mathcal{S}^{\perp T} \mathcal{L} \mathcal{S}^{\perp} < 0$
- $\exists \theta \in \mathbb{R} : \mathcal{L} - \theta \mathcal{S}^T \mathcal{S} < 0$
- $\exists \mathcal{D} \in \mathbb{R}^{m \times p} : \mathcal{L} + \mathcal{D}\mathcal{S} + \mathcal{S}^T \mathcal{D}^T < 0$

Lemma 2.3 *Error system (10) is stable and FF in (15) is fulfilled, on condition that there are $\mathcal{B}$, $0 < C$, satisfying*

$$
\begin{bmatrix} \bar{M}(l, \hat{l}) & \bar{N}(l, \hat{l}) \\ I & 0 \end{bmatrix}^T \begin{bmatrix} \mathcal{B} & e^{j\tilde{\varrho}_3} C \\ e^{-j\tilde{\varrho}_3} C & -\mathcal{B} - 2cos(\tilde{\varrho}_4) C \end{bmatrix} \begin{bmatrix} \bar{M}(l, \hat{l}) & \bar{N}(l, \hat{l}) \\ I & 0 \end{bmatrix} + \begin{bmatrix} \bar{J}^T(l, \hat{l}) \bar{J}(l, \hat{l}) & 0 \\ 0 & -\beta^2 I \end{bmatrix} < 0 \tag{16}
$$

with $\quad \tilde{\varrho}_3 = \frac{\tilde{\varrho}_2 + \tilde{\varrho}_1}{2}$, $\tilde{\varrho}_4 = \frac{\tilde{\varrho}_2 - \tilde{\varrho}_1}{2}$.

Proof 2.4 *The substantiation of Lemma 2.3 is defined in appendix.*

3 FF performance analysis

Theorem 3.1 *Error model (10) is stable, FF index (15) is fulfilled, on condition that there are $\mathcal{X}$, $\mathcal{B}$, C, $\mathcal{E}$, $\mathcal{Z}$, $\mathcal{T}$ satisfying $C > 0$, $\mathcal{X} > 0$ and*

$$
\begin{bmatrix} \daleth_1 & \daleth_2 & \mathcal{Z}\bar{N}(l, \hat{l}) - \mathcal{T}^T & 0 \\ * & \daleth_3 & \daleth_4 & \bar{J}^T(l, \hat{l}) \\ * & * & \daleth_5 & 0 \\ * & * & * & -I \end{bmatrix} < 0 \tag{17}
$$

$$
\begin{bmatrix} \mathcal{X} - \mathcal{Z} - \mathcal{Z}^T & \mathcal{Z}M(l, \hat{l}) - \mathcal{E}^T \\ * & \Psi_1 \end{bmatrix} < 0 \tag{18}
$$

with

$$
\begin{aligned}
\daleth_1 &= \mathcal{B} - \mathcal{Z} - \mathcal{Z}^T; \\
\daleth_2 &= e^{j\tilde{\varrho}_3} C - \mathcal{E}^T + \mathcal{Z}\bar{A}(l, \hat{l}); \\
\daleth_3 &= -\mathcal{B} - 2cos(\tilde{\varrho}_4) C + \mathcal{E}\bar{M}(l, \hat{l}) + \bar{M}^T(l, \hat{l}) \mathcal{E}^T; \\
\daleth_4 &= \mathcal{E}\bar{N}(l, \hat{l}) + \bar{M}^T(l, \hat{l}) l^T(l) \\
\daleth_5 &= -\beta^2 I + \mathcal{T}\bar{N}(l, \hat{l}) + \bar{N}^T(l, \hat{l}) l^T; \\
\triangle_1 &= -\mathcal{X} + \mathcal{E}\bar{M}(l, \hat{l}) + \bar{M}^T(l, \hat{l}) \mathcal{E}^T.
\end{aligned}
$$

Proof 3.2 *First, Condition (17) maybe written as*

$$
\begin{pmatrix} \bar{\boldsymbol{M}}(\boldsymbol{l},\hat{\boldsymbol{l}}) & \bar{\boldsymbol{N}}(\boldsymbol{l},\hat{\boldsymbol{l}}) \\ I & 0 \\ 0 & I \end{pmatrix}^T [\begin{pmatrix} 0 & \bar{\boldsymbol{J}}(\boldsymbol{l},\hat{\boldsymbol{l}}) & 0 \\ 0 & 0 & I \end{pmatrix}^T
\begin{pmatrix} I & 0 \\ 0 & -\beta^2 I \end{pmatrix} \begin{pmatrix} 0 & \bar{\boldsymbol{J}}(\boldsymbol{l},\hat{\boldsymbol{l}}) & 0 \\ 0 & 0 & I \end{pmatrix} \begin{pmatrix} I & 0 \\ 0 & I \\ 0 & 0 \end{pmatrix}
\begin{pmatrix} \mathcal{B} & e^{j\varrho_c} C \\ e^{-j\tilde{\varrho}_3} C & -\mathcal{B} - 2cos(\tilde{\varrho}_4) C \end{pmatrix} \begin{pmatrix} I & 0 & 0 \\ 0 & I & 0 \end{pmatrix}]
\begin{pmatrix} \bar{\boldsymbol{M}}(\boldsymbol{l},\hat{\boldsymbol{l}}) & \bar{\boldsymbol{N}}(\boldsymbol{l},\hat{\boldsymbol{l}}) \\ I & 0 \\ 0 & I \end{pmatrix} < 0 \tag{19}
$$

Denote

$$
\begin{aligned}
\mathcal{D} &= \begin{bmatrix} \mathcal{E} \\ \mathcal{Z} \\ \mathcal{T} \end{bmatrix}; \quad \theta = \begin{bmatrix} \nabla(\mu+1) \\ \nabla(\mu) \\ \boldsymbol{d}(\mu) \end{bmatrix}; \\
\mathcal{S} &= \begin{bmatrix} -I & \bar{\boldsymbol{M}}(\boldsymbol{l},\hat{\boldsymbol{l}}) & \bar{\boldsymbol{N}}(\boldsymbol{l},\hat{\boldsymbol{l}}) \end{bmatrix}
\end{aligned} \tag{20}
$$

With the help of Lemma 2.2, we have

$$
\mathcal{D}\mathcal{S} + \mathcal{S}^T\mathcal{D}^T + \begin{bmatrix} 0 & \bar{\boldsymbol{J}}(\boldsymbol{l},\hat{\boldsymbol{l}}) & 0 \\ 0 & 0 & I \end{bmatrix}^T
\begin{bmatrix} I & 0 \\ 0 & -\beta^2 I \end{bmatrix} \begin{bmatrix} 0 & \bar{\boldsymbol{J}}(\boldsymbol{l},\hat{\boldsymbol{l}}) & 0 \\ 0 & 0 & I \end{bmatrix} \begin{bmatrix} I & 0 \\ 0 & I \\ 0 & 0 \end{bmatrix}
\begin{bmatrix} \mathcal{B} & e^{j\tilde{\varrho}_3} C \\ e^{-j\tilde{\varrho}_3} C & -\mathcal{B} - 2cos(\tilde{\varrho}_4) C \end{bmatrix} \begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \end{bmatrix} < 0 \tag{21}
$$

With the help of Lemma 2.2, given (16).
Consider the Lyapunov equation :

$$
\begin{bmatrix} \bar{\boldsymbol{M}}(\boldsymbol{l},\hat{\boldsymbol{l}}) \\ I \end{bmatrix}^T \begin{bmatrix} \mathcal{X} & 0 \\ 0 & -\mathcal{X} \end{bmatrix} \begin{bmatrix} \bar{\boldsymbol{M}}(\boldsymbol{l},\hat{\boldsymbol{l}}) \\ I \end{bmatrix} < 0 \tag{22}
$$

Let

$$
\begin{aligned}
\theta &= \begin{bmatrix} \nabla(\mu+1) \\ \nabla(\mu) \end{bmatrix}; \quad \mathcal{L} = \begin{bmatrix} \mathcal{X} & 0 \\ 0 & -\mathcal{X} \end{bmatrix}; \\
\mathcal{S} &= \begin{bmatrix} -I & \bar{\boldsymbol{M}}(\boldsymbol{l},\hat{\boldsymbol{l}}) \end{bmatrix}; \quad \mathcal{D} = \begin{bmatrix} \mathcal{E}^T & \mathcal{Z}^T \end{bmatrix}^T
\end{aligned} \tag{23}
$$

Using the conditions in 2.2, then

$$
\begin{bmatrix} \mathcal{X} & 0 \\ 0 & -\mathcal{X} \end{bmatrix} + \begin{bmatrix} \mathcal{E} \\ \mathcal{Z} \end{bmatrix} \begin{bmatrix} -I & \bar{\boldsymbol{M}}(\boldsymbol{l},\hat{\boldsymbol{l}}) \end{bmatrix} +
\begin{bmatrix} -I & \bar{\boldsymbol{M}}(\boldsymbol{l},\hat{\boldsymbol{l}}) \end{bmatrix}^T \begin{bmatrix} \mathcal{E} \\ \mathcal{Z} \end{bmatrix}^T < 0 \tag{24}
$$

which is nothing but (18).

4 FF performance Design

Theorem 4.1 *Error model (10) is stable, FF index (15) is fulfilled, on condition that there are* $\mathcal{B} = \begin{bmatrix} \mathcal{B}_1 & \mathcal{B}_2 \\ * & \mathcal{B}_3 \end{bmatrix}$, $C = \begin{bmatrix} C_1 & C_2 \\ * & C_3 \end{bmatrix} > 0$, $\mathcal{X} = \begin{bmatrix} \mathcal{X}_1 & \mathcal{X}_2 \\ * & \mathcal{X}_3 \end{bmatrix} > 0$, $\check{\boldsymbol{M}}(\hat{\boldsymbol{l}})$, $\check{\boldsymbol{N}}(\hat{\boldsymbol{l}})$, $\check{\boldsymbol{J}}(\hat{\boldsymbol{l}})$, $\mathcal{E}_1$, $\mathcal{E}_2$, $\mathcal{Z}_1$, $\mathcal{Z}_2$, $\mathcal{T}_1$, $\mathcal{V}$, *satisfying*

$$
\begin{bmatrix} \bar{\Delta}_{11} & \bar{\Delta}_{12} & \bar{\Delta}_{13} & -\mathcal{Z}_2^T + E\hat{\boldsymbol{M}}(\hat{\boldsymbol{l}}) \\ * & \bar{\Delta}_{22} & \bar{\Delta}_{23} & \hat{\boldsymbol{M}}(\hat{\boldsymbol{l}}) \\ * & * & \bar{\Delta}_{33} & -\mathcal{X}_2 + \boldsymbol{M}^T(\boldsymbol{l})\mathcal{Z}_2^T \\ * & * & * & -\mathcal{X}_3 \end{bmatrix} < 0 \tag{25}
$$

$$
\begin{bmatrix} \bar{\daleth}_{11} & \bar{\daleth}_{12} & \bar{\daleth}_{13} & \bar{\daleth}_{14} & \bar{\daleth}_{15} & 0 \\ * & \bar{\daleth}_{22} & \bar{\daleth}_{23} & \bar{\daleth}_{24} & \bar{\daleth}_{25} & 0 \\ * & * & \bar{\daleth}_{33} & \bar{\daleth}_{34} & \bar{\daleth}_{35} & \boldsymbol{J}^T(\boldsymbol{l}) \\ * & * & * & \bar{\daleth}_{44} & \bar{\daleth}_{45} & -\hat{\boldsymbol{J}}^T(\hat{\boldsymbol{l}}) \\ * & * & * & * & \bar{\daleth}_{55} & 0 \\ * & * & * & * & * & -I \end{bmatrix} < 0 \tag{26}
$$

with

$$
\begin{aligned}
\bar{\Delta}_{11} &= \mathcal{B}_1 - \mathcal{E}_1 - \mathcal{E}_1^T; \ \bar{\Delta}_{12} = \mathcal{X}_2 - E\mathcal{V} - \mathcal{E}_2^T; \\
\bar{\Delta}_{13} &= \mathcal{E}_1\boldsymbol{A}(h) - \mathcal{Z}_1^T; \ \bar{\Delta}_{22} = \mathcal{X}_3 - \mathcal{V} - \mathcal{V}^T; \\
\bar{\Delta}_{23} &= \mathcal{E}_2\boldsymbol{M}(\boldsymbol{l}); \ \ \bar{\Delta}_{33} = -\mathcal{X}_1 + \mathcal{Z}_1\boldsymbol{M}(\boldsymbol{l}) + \boldsymbol{M}^T(\boldsymbol{l})\mathcal{Z}_1^T;
\end{aligned}
$$

$$
\begin{aligned}
\bar{\daleth}_{11} &= \mathcal{B}_1 - \mathcal{E}_1 - \mathcal{E}_1^T; \ \bar{\daleth}_{12} = \mathcal{B}_2 - E\mathcal{V} - \mathcal{E}_2^T; \\
\bar{\daleth}_{13} &= e^{j\tilde{\varrho}_3}C_1 - \mathcal{Z}_1^T + \mathcal{E}_1A(\boldsymbol{l}); \ \bar{\daleth}_{22} = \mathcal{B}_3 - \mathcal{V} - \mathcal{V}^T; \\
\bar{\daleth}_{14} &= e^{j\tilde{\varrho}_3}C_2 - \mathcal{Z}_2^T + E\tilde{\mathcal{M}}(\hat{\boldsymbol{l}}); \\
\bar{\daleth}_{24} &= e^{j\tilde{\varrho}_3}C_3 + \tilde{\mathcal{M}}(\hat{\boldsymbol{l}}); \\
\bar{\daleth}_{15} &= \mathcal{E}_1\boldsymbol{N}(\boldsymbol{l}) + E\tilde{N}(\hat{\boldsymbol{l}}) - \mathcal{T}_1^T; \\
\bar{\daleth}_{25} &= \mathcal{Z}_2\boldsymbol{N}(\boldsymbol{l}) + \tilde{\boldsymbol{M}}(\hat{\boldsymbol{l}}); \\
\bar{\daleth}_{33} &= -\mathcal{B}_1 - 2cos(\tilde{\varrho}_4)C_1 + \mathcal{Z}_1\boldsymbol{M}(\boldsymbol{l}) + \boldsymbol{M}^T(\boldsymbol{l})\mathcal{Z}_1^T; \\
\bar{\daleth}_{34} &= -\mathcal{B}_2 - 2cos(\tilde{\varrho}_4)C_2 + \boldsymbol{M}^T(\boldsymbol{l})\mathcal{Z}_2^T; \\
\bar{\daleth}_{35} &= \mathcal{Z}_1\boldsymbol{N}(\boldsymbol{l}) + \boldsymbol{M}^T(\boldsymbol{l})\mathcal{T}_1^T; \\
\bar{\daleth}_{23} &= e^{-j\tilde{\varrho}_3}C_2 + \mathcal{E}_2\boldsymbol{M}(\boldsymbol{l}); \\
\bar{\daleth}_{44} &= -\mathcal{B}_3 - 2cos(\tilde{\varrho}_4)C_3; \ \bar{\daleth}_{45} = \mathcal{Z}_2\boldsymbol{N}(\boldsymbol{l}); \\
\bar{\daleth}_{55} &= -\beta^2 I + sym[\mathcal{T}_1\boldsymbol{N}(\boldsymbol{l})]
\end{aligned}
$$

The following parameters as

$$
\begin{aligned}
\hat{\boldsymbol{M}}(\hat{\boldsymbol{l}}) &= \mathcal{V}^{-1}\check{\boldsymbol{M}}(\hat{h});; \\
\hat{\boldsymbol{N}}(\hat{\boldsymbol{l}}) &= \mathcal{V}^{-1}\check{\boldsymbol{N}}(\hat{h}); \\
\hat{\boldsymbol{J}}(\hat{\boldsymbol{l}}) &= \check{\boldsymbol{J}}(\hat{\boldsymbol{l}})
\end{aligned} \tag{27}
$$

Proof 4.2 *Parameterise slack matrices* $\mathcal{E}$, $\mathcal{Z}$, $\mathcal{T}$ *in Theorem 3.1 as*

$$
\begin{aligned}
\mathcal{E} &= \begin{bmatrix} \mathcal{E}_1 & E\mathcal{V} \\ \mathcal{E}_2 & \mathcal{V} \end{bmatrix}; \quad \mathcal{Z} = \begin{bmatrix} \mathcal{Z}_1 & 0 \\ \mathcal{Z}_2 & 0 \end{bmatrix}; \\
\mathcal{T} &= \begin{bmatrix} \mathcal{T}_1 & 0 \end{bmatrix}
\end{aligned} \tag{28}
$$

where $E = \begin{pmatrix} I \\ 0 \end{pmatrix}$.

Moreover, the Theorem 4.3 is to solve the FFMR problem in the FF index (15).

Theorem 4.3 *Error model (10) is stable, FF index (15) is fulfilled, on condition that there are* $\mathcal{B} = \begin{bmatrix} \mathcal{B}_1 & \mathcal{B}_2 \\ * & \mathcal{B}_3 \end{bmatrix}$, $\mathcal{C} = \begin{bmatrix} \mathcal{C}_1 & \mathcal{C}_2 \\ * & \mathcal{C}_3 \end{bmatrix} > 0$, $\mathcal{X} = \begin{bmatrix} \mathcal{X}_1 & \mathcal{X}_2 \\ * & \mathcal{X}_3 \end{bmatrix} > 0$, $\breve{\boldsymbol{M}}_x$, $\breve{\boldsymbol{N}}_x$, $\breve{\boldsymbol{J}}_x$, $\mathcal{E}_1$, $\mathcal{E}_2$, $\mathcal{Z}_1$, $\mathcal{Z}_2$, $\mathcal{T}_1$, $\mathcal{V}$, *satisfying, with* $x, t \in \{1, 2, ..., m\}$

$$\begin{bmatrix} \tilde{\Delta}_{11} & \tilde{\Delta}_{12} & \tilde{\Delta}_{13} & -\mathcal{Z}_2^T + E\breve{\mathcal{A}}_x \\ * & \tilde{\Delta}_{22} & \tilde{\Delta}_{23} & \breve{\mathcal{A}}_x \\ * & * & \tilde{\Delta}_{33} & -\mathcal{X}_2 + A_t^T \mathcal{Z}_2^T \\ * & * & * & -\mathcal{X}_3 \end{bmatrix} < 0 \tag{29}$$

$$\begin{bmatrix} \tilde{\daleth}_{11} & \tilde{\daleth}_{12} & \tilde{\daleth}_{13} & \tilde{\daleth}_{14} & \tilde{\daleth}_{15} & 0 \\ * & \tilde{\daleth}_{22} & \tilde{\daleth}_{23} & \tilde{\daleth}_{24} & \tilde{\daleth}_{25} & 0 \\ * & * & \tilde{\daleth}_{33} & \tilde{\daleth}_{34} & \tilde{\daleth}_{35} & \boldsymbol{J}_t^T \\ * & * & * & \tilde{\daleth}_{44} & \tilde{\daleth}_{45} & -\breve{\boldsymbol{J}}_x^T \\ * & * & * & * & \tilde{\daleth}_{55} & 0 \\ * & * & * & * & * & -I \end{bmatrix} < 0 \tag{30}$$

with

$$\begin{aligned}
\tilde{\Delta}_{11} &= \mathcal{B}_1 - \mathcal{E}_1 - \mathcal{E}_1^T;\ \tilde{\Delta}_{12} = \mathcal{X}_2 - E\mathcal{V} - \mathcal{E}_2^T; \\
\tilde{\Delta}_{13} &= \mathcal{E}_1 \boldsymbol{M}_t - \mathcal{Z}_1^T; \\
\tilde{\Delta}_{22} &= \mathcal{X}_3 - \mathcal{V} - \mathcal{V}^T; \\
\tilde{\Delta}_{23} &= \mathcal{E}_2 \boldsymbol{M}_t;\ \tilde{\Delta}_{33} = -\mathcal{X}_1 + \mathcal{Z}_1 \boldsymbol{M}_t + \boldsymbol{M}_t^T \mathcal{Z}_1^T; \\
\tilde{\daleth}_{11} &= \mathcal{B}_1 - \mathcal{E}_1 - \mathcal{E}_1^T;\ \tilde{\daleth}_{12} = \mathcal{B}_2 - E\mathcal{V} - \mathcal{E}_2^T; \\
\tilde{\daleth}_{13} &= e^{j\tilde{\varrho}_3}\mathcal{C}_1 - \mathcal{Z}_1^T + \mathcal{E}_1 \boldsymbol{M}_t;\ \tilde{\daleth}_{22} = \mathcal{B}_3 - \mathcal{V} - \mathcal{V}^T; \\
\tilde{\daleth}_{14} &= e^{j\tilde{\varrho}_3}\mathcal{C}_2 - \mathcal{Z}_2^T + E\breve{\boldsymbol{M}}_x; \\
\tilde{\daleth}_{24} &= e^{j\tilde{\varrho}_3}\mathcal{C}_3 + \breve{\boldsymbol{M}}_x; \\
\tilde{\daleth}_{15} &= \mathcal{E}_1 \boldsymbol{N}_t + E\breve{\boldsymbol{N}}_x - \mathcal{T}_1^T;\ \tilde{\daleth}_{25} = \mathcal{Z}_2 \boldsymbol{N}_t + \breve{\boldsymbol{M}}_x; \\
\tilde{\daleth}_{33} &= -\mathcal{B}_1 - 2cos(\tilde{\varrho}_4)\mathcal{C}_1 + \mathcal{Z}_1 \boldsymbol{M}_t + \boldsymbol{M}_t^T \mathcal{Z}_1^T; \\
\tilde{\daleth}_{34} &= -\mathcal{B}_2 - 2cos(\tilde{\varrho}_4)\mathcal{C}_2 + \boldsymbol{M}_t^T \mathcal{Z}_2^T; \\
\tilde{\daleth}_{35} &= \mathcal{Z}_1 \boldsymbol{N}_t + \boldsymbol{M}_t^T \mathcal{T}_1^T;\ \tilde{\daleth}_{23} = e^{-j\tilde{\varrho}_3}\mathcal{C}_2 + \mathcal{E}_2 \boldsymbol{M}_t; \\
\tilde{\daleth}_{44} &= -\mathcal{B}_3 - 2cos(\tilde{\varrho}_4)\mathcal{C}_3;\ \tilde{\daleth}_{45} = \mathcal{Z}_2 \boldsymbol{N}_t; \\
\tilde{\daleth}_{55} &= -\beta^2 I + sym[\mathcal{T}_1 \boldsymbol{N}_t].
\end{aligned}$$

The following reduced-order parameters as :

$$\begin{aligned}
\hat{\mathbf{M}}_t &= \mathcal{V}^{-1}\breve{\mathbf{M}}_x \\
\hat{\mathbf{N}}_t &= \mathcal{V}^{-1}\breve{\mathbf{N}}_x; \\
\hat{\mathbf{J}}_t &= \breve{\mathbf{J}}_x
\end{aligned} \tag{31}$$

Proof 4.4 *We propose the following equations :*

$$\begin{aligned}
&\sum_{x=1}^{q}\sum_{t=1}^{m} \hat{\boldsymbol{l}}_x \boldsymbol{l}_t \bar{\Delta}_{xt} \\
&\sum_{x=1}^{q}\sum_{t=1}^{m} \hat{\boldsymbol{l}}_x \boldsymbol{l}_t \bar{\daleth}_{xt};
\end{aligned} \tag{32}$$

Therefore, us find Theorem 4.1.

Remark 4.5 *If us pick* $\mathcal{C} = 0$, *we mastery employ theorem 4.3 to settle the* H_∞ *MR nonlinear systems over EF range.*

5 Numerical Example

Consider a fuzzy system in discrete time, represents in [12] :

$$\begin{aligned}
\mathbf{J}_1 &= \begin{bmatrix} 1.4419 & 0.6720 & 0.1387 & -0.8595 \end{bmatrix} \\
\mathbf{J}_2 &= \begin{bmatrix} 1.3329 & 0.6720 & 0.1387 & -0.8478 \end{bmatrix}; \\
\mathbf{M}_1 &= \begin{bmatrix} 0.1612 & 0.0574 & -0.0144 & 0.1846 \\ 0.0434 & -0.3638 & 0.5258 & -0.0357 \\ -0.0747 & -0.3146 & -0.0487 & -0.1043 \\ -0.1664 & 0.4031 & 0.0347 & 0.2864 \end{bmatrix}; \\
\mathbf{M}_2 &= \begin{bmatrix} 0.1312 & 0.0474 & -0.0044 & 0.1546 \\ 0.0234 & -0.3018 & 0.4258 & -0.0357 \\ -0.0554 & -0.2421 & -0.0367 & -0.0843 \\ -0.1551 & 0.3031 & 0.0247 & 0.1864 \end{bmatrix}; \\
\mathbf{N}_1 &= \begin{bmatrix} 0.2023 \\ -0.2313 \\ -0.1137 \\ 0.1279 \end{bmatrix};\ \mathbf{N}_2 = \begin{bmatrix} 0.0123 \\ -0.1313 \\ -0.1138 \\ 0.1179 \end{bmatrix}; \\
E &= \begin{bmatrix} I & 0 \end{bmatrix}.
\end{aligned} \tag{33}$$

The normalized membership function :

$$\begin{aligned}
\mathbf{T}_1(\mathbf{z}_1(\mu)) &= \frac{1 - \sin(\mathbf{z}_1(\mu))}{2} \\
\mathbf{T}_2(\mathbf{z}_1(\mu)) &= 1 - \mathbf{T}_1(\mathbf{z}_1(\mu))
\end{aligned} \tag{34}$$

Via using Theorem 4, the obtained matrix parameters of FF H_∞ reduced order systems are the following :

- EF area : $(-\pi \leq \tilde{\varrho} \leq \pi)$

$$\begin{aligned}
\left[\begin{array}{c|c} \hat{\mathbf{M}}_1 & \hat{\mathbf{N}}_1 \\ \hline \hat{\mathbf{J}}_1 & - \end{array}\right] &= \left[\begin{array}{cc|c} -0.7415 & -0.7001 & -0.1925 \\ 0.7145 & 0.1925 & 0.4512 \\ \hline -0.8874 & -1.2145 & - \end{array}\right]; \\
\left[\begin{array}{c|c} \hat{\mathbf{M}}_2 & \hat{\mathbf{N}}_2 \\ \hline \hat{\mathbf{J}}_2 & - \end{array}\right] &= \left[\begin{array}{cc|c} -0.8840 & 0.7415 & 0.1325 \\ -0.6245 & 0.2415 & -0.4125 \\ \hline -1.2021 & -1.1325 & - \end{array}\right]
\end{aligned} \tag{35}$$

- FF area : $(\frac{\pi}{7} \leq \tilde{\varrho} \leq \frac{\pi}{4})$

$$\left[\begin{array}{c|c} \hat{\mathbf{M}}_1 & \hat{\mathbf{N}}_1 \\ \hline \hat{\mathbf{J}}_1 & - \end{array}\right] = \left[\begin{array}{cc|c} -0.4415 & 0.2145 & -0.1125 \\ -0.1745 & 0.1325 & 0.1954 \\ \hline -0.2314 & 0.7458 & - \end{array}\right];$$
$$\left[\begin{array}{c|c} \hat{\mathbf{M}}_2 & \hat{\mathbf{N}}_2 \\ \hline \hat{\mathbf{J}}_2 & - \end{array}\right] = \left[\begin{array}{cc|c} -0.6614 & 0.2845 & -0.1354 \\ -0.3325 & 0.1325 & 0.2157 \\ \hline -0.3458 & 0.5842 & - \end{array}\right] \tag{36}$$

The comparison result with the technique proposed in Theorem 4.3 illustrate in Table 1, that indicate the little conservation of the method suggest in the paper.

The initial cases are proposed null. Figure 1 illustrate the errors $e(k)$ from various approaches. It illustrate the FF method have the best performance in comparison with the EF approach.

Table 1: H_∞ performance apply from various approaches

Frequency	*Methods*	β_{min}	*Max error*
$-\pi \le \varrho \le \pi$	[12]	0.9715	0.0915
$-\pi \le \varrho \le \pi$	**Th 4.3** (Q=0)	0.5514	0.0423
$\frac{\pi}{7} \le \varrho \le \frac{\pi}{3}$	**Th 4.3**	**0.1305**	**0.0088**

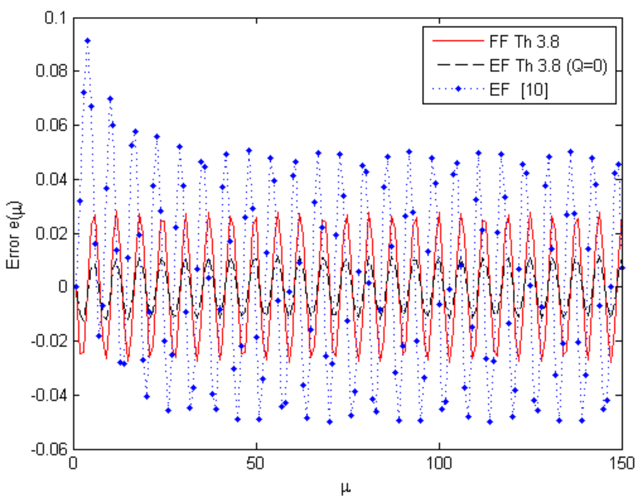

Figure 1: Error response of $\mathbf{g}_\mu$.

The ratio $\mathbf{W}(\mu)$ is presented as :

$$\mathbf{W}(\mu) = \sqrt{\sum_{\mu=0}^{\infty} \mathbf{g}^T(\mu)\mathbf{g}(\mu) / \sum_{\mu=0}^{\infty} \mathbf{d}^T(\mu)\mathbf{d}(\mu)} \tag{37}$$

Figures 2 indicate the the ratio in (37), we could notice whether the error model is stable, knowing that the initial Condition are null, that indicate the little conservation of the method suggest in the paper.

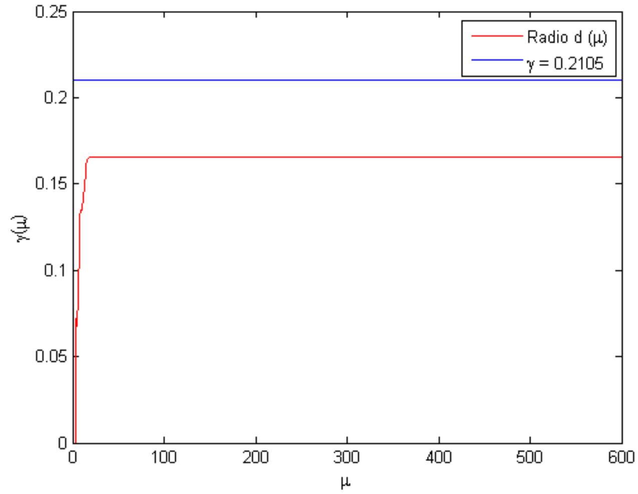

Figure 2: Value of $\mathbf{W}(\mu)$.

6 Conclusion

This work, we dealt with problematic for the FF model reduction design of nonlinear systems over FF ranges. We have suggested a model reduction process in order to minimize the conservatism design using the frequency information of the disturbances and we have assumed that the disturbances are known in a recognized FF domain. Also, systematic techniques have been suggested for the generation of a model reduction which ensures asymptotic stability and FF H_∞ index, at the basis of a more general linearization procedure.

References

[1] T. Takagi, M. Sugeno, " Fuzzy identification of systems and its applications to modeling and control," IEEE transactions on systems, man, and cybernetics, SMC-**15**(1), 116-132, 1985, doi: 10.1109/TSMC.1985.6313399

[2] H. Zhang, Y. Shi, A. S. Mehr, "On H_∞ filtering for discrete time Takagi Sugeno fuzzy Systems," IEEE Transactions on Fuzzy Systems, **20**(2), 396-401, 2011, doi: 10.1109/TFUZZ.2011.2175933

[3] A. Benzaouia, A. Hmamed, A. EL Hajjaji, " Stabilization of controlled positive discrete-time T-S fuzzy systems by state feedback control," International Journal of Adaptive Control and Signal Processing, **24**(12), 1091-1106, 2010, doi: 10.1002/acs.1185

[4] H. Gao, Y. Zhao, J. Lam, K. Chen, "H_∞ fuzzy filtering of nonlinear systems with intermittent measurements," IEEE Transactions on Fuzzy Systems, **17**(2), 289-299, 2009.

[5] L. Zhang, H. Yang, Q. Mao, "T S fuzzy system identification based on bayesian inference," ICIC Express Letter, **13**(1), 51-58, 2019.

[6] M. Nentwig, P. Mercorelli. "hrottle valve control using an inverse local linear model tree based on a fuzzy neural network," 7th International Conference on Cybernetic Intelligent Systems, IEEE, 1-6, 2008, doi: 10.1109/UKRICIS.2008.4798943

[7] M. Schimmack, F. Pöschke, H., Schulte, P. Mercorelli, "Observer Strategies for Virtual Sensing of Embroidered Metal-Polymer Heater Structure," 23rd International Conference on Methods & Models in Automation & Robotics (MMAR) IEEE, 887-892, 2018, doi: 10.1109/MMAR.2018.8486033

[8] A. C. Antoulas, "An overview of approximation methods for large-scale dynamical systems," Annual reviews in Control, **29**(2), 181-190, 2005, doi: 10.1016/j.arcontrol.2005.08.002

[9] K. Glover, "All optimal Hankel-norm approximations of linear multivariable systems and their $L,^{\infty}$-error bounds". International journal of control," **39**(6), 1115-1193, 1984, doi: 10.1080/00207178408933239

[10] G. Lastman, "Reduced-order aggregated models for bilinear time-invariant dynamical systems," IEEE transactions on automatic control, **29**(4), 359-361, 1984, doi: 10.1109/TAC.1984.1103534

[11] B. Moore, "Principal component analysis in linear systems: Controllability, observability, and model reduction," IEEE transactions on automatic control, **26**(1), 17-32, 1981, doi: 10.1109/TAC.1981.1102568

[12] L. Wu, X. Su, J. Qiu, "Model approximation for discrete-time state-delay systems in the T–S fuzzy framework," IEEE Transactions on Fuzzy Systems, **19**(2), 366-378, 2011, doi: 10.1109/TFUZZ.2011.2104363

[13] D. Wang, A. Zilouchian. "Model reduction of discrete linear systems via frequency-domain balanced structure," IEEE Transactions on Circuits and Systems I: Fundamental Theory and Applications, **47**(6), 830-837, 2000, doi: 10.1109/81.852936

[14] X. Du, G. H. Yang, "H_∞ model reduction of linear continuous-time sys-tems over finite-frequency interval," IET control theory & applications, **4**(3), 499-508, 2020, doi : 10.1049/iet-cta.2008.0537

[15] X. Li, S. Yin, H. Gao, "Passivity-preserving model reduction with finite frequency H_∞ approximation performance," Automatica, **50**(9), 2294-2303, 2014, doi: 10.1016/j.automatica.2014.07.001

[16] D. W. Ding, X. J. Li, X. Du, X. Xie," Finite-frequency model reduction of Takagi-Sugeno fuzzy systems," IEEE Transactions on Fuzzy Systems, **24**(6), 1464-1474, 2016, doi: 10.1109/TFUZZ.2016.2540060

[17] A. El-Amrani, B. Boukili, A. El-Hajjaji, A. Hmamed, "H_∞ model reduc-tion for T-S fuzzy systems over finite frequency ranges," optimal control applications and methods, **39**(4), 1479-1496, 2018, doi : 10.1002/oca.2422

[18] D. W. Ding, X. Xie, X. Du, X. J. Li, "Finite-frequency model reduction of discrete-time T–S fuzzy state-delay systems," Neurocomputing, **203**, 121-128, 2016, doi: https://doi.org/10.1016/j.neucom.2016.03.053

[19] A. El-Amrani, A. El Hajjaji, I. Boumhidi, A. Hmamed, A. Aitouche, "Finite Frequency Approach for H_∞ model reduction of 2D continuous systems," 27th Mediterranean Conference on Control and Automation (MED) IEEE, 177-182, 2019, doi: 10.1109/MED.2019.8798569

[20] X. Su, L. Wu, P. Shi, Y. D. Song, "H_∞ model reduction of Takagi-Sugeno fuzzy stochastic systems", IEEE transactions on systems, man, and cybernet-ics. Part B, Cybernetics: a publication of the IEEE Systems, Man, and Cyber-netics Society, **42**(6), 1574-1585, 2012, doi: 10.1109/tsmcb.2012.2195723

[21] A. V. Oppenheim, A. S. Willsky, S. H. Nawab. "Signals and Systems", 2nd Edition,. Prentice Hall, Boston University, 1-987, 1996.

[22] M. J., Lacerda, R. C. F., Oliveira, P. L. D., Peres, "Robust H_2 and H_∞ filter design for uncertain linear systems via LMIs and polynomial matrices," Signal Processing, **91**(5), 1115-1122, 2011, doi: 10.1016/j.sigpro.2010.10.013

7 Appendix

Multiply equation (16) of $\begin{bmatrix} \nabla(\mu) \\ \mathbf{d}(\mu) \end{bmatrix}$ for right and by its transposition to the left, we get

$$\begin{aligned}
&\beta^2 \mathbf{d}(\mu)^T \mathbf{d}(\mu) + tr(C(\mathbf{g}^{j\tilde{\varrho}_3}\nabla(\mu)\nabla(\mu+1)^T \\
-\ & e^{-j\tilde{\varrho}_3}\nabla(\mu+1)\nabla(\mu)^T - 2cos(\tilde{\varrho}_4)\nabla(\mu)\nabla(\mu)^T)) \\
-\ & \nabla^T(\mu+1)\mathcal{B}\nabla(\mu+1) - \nabla^T(\mu)\mathcal{B}\nabla(\mu) + \mathbf{g}^T(\mu)\mathbf{g}(\mu) \leq 0
\end{aligned} \tag{38}$$

By taking the sum of $\mu = 0$ in π,seeing that $\nabla(0) = 0$ and $\lim_{\mu\to pi}\nabla(\mu) = 0$ we get

$$\begin{aligned}
&\sum_{\mu=0}^{\pi}\mathbf{g}^T(\mu)\mathbf{g}(\mu) - \beta^2\sum_{\mu=0}^{\pi}\mathbf{d}(\mu)^T\mathbf{d}(\mu) \\
+\ & Tr[C\sum_{\mu=0}^{\pi}(e^{j(\tilde{\varrho}_3}\nabla(\mu)\nabla(\mu+1)^T + e^{-j\tilde{\varrho}_3}\nabla(\mu+1)\nabla(\mu)^T \\
-\ & 2\cos(\tilde{\varrho}_4)\nabla(\mu)\nabla(\mu)^T)] \leq 0
\end{aligned} \tag{39}$$

From Parseval function [21], we get :

$$\begin{aligned}
&\beta^2\frac{1}{2\pi}\int_{-\pi}^{\pi}\mathbf{d}^T(\tilde{\varrho})\mathbf{d}(\tilde{\varrho})d\tilde{\varrho} \\
-\ & Tr[\frac{\cos(\tilde{\varrho}-\tilde{\varrho}_c)-\cos\tilde{\varrho}_w}{\pi}\int_{-\pi}^{\pi}\mathcal{M}^T(\tilde{\varrho})C\mathcal{M}(\tilde{\varrho})d\tilde{\varrho}] \\
-\ & \frac{1}{2\pi}\int_{-\pi}^{\pi}\mathbf{g}^T(\tilde{\varrho})\mathbf{g}(\tilde{\varrho})d\tilde{\varrho} \geq 0
\end{aligned} \tag{40}$$

Then :

$$\cos(\tilde{\varrho}-\tilde{\varrho}_3) - \cos(\tilde{\varrho}_4) \geq 0 \tag{41}$$

have toward all gifted $\tilde{\varrho}_1 \leq \tilde{\varrho} \leq \tilde{\varrho}_2$. From $C > 0$ and (41), it attends that

$$Tr[\frac{\cos(\tilde{\varrho}-\tilde{\varrho}_c)-\cos(\tilde{\varrho}_4)}{\pi}\int_{-\pi}^{\pi}\mathcal{M}^T(\tilde{\varrho})C\mathcal{M}(\tilde{\varrho})d\tilde{\varrho}] \geq 0 \tag{42}$$

seeing that, (40) and the FF of signal input $\tilde{\varrho}_1 \leq \tilde{\varrho} \leq \tilde{\varrho}_2$, thus (15) is fulfilled.