Lacol Interpolation Bicubic Spline Method in Digital Processing of Geophysical Signals

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A R T I C L E   I N F O

A B S T R A C T

The paper a cubic spline built through a local base spline and the local interpolation bicubic spline models we offer have been selected. The construction details of the models are given, the two-dimensional local interpolation bicubic spline models considered in this study provide high accuracy in digital processing of signals, which helps experts to make the right decision as a result of digital processing of signals. As an example, the initial values of the geophysical signal were digitally processed and error results were obtained. The error results obtained by digital processing of the geophysical signal of the considered models were compared on the basis of numerical and graphical comparisons.

1. Introduction

In many geophysical surveys, scientists have sought to find predators that provide information about the location of minerals. An anomalous change in a parameter is a sudden increase or decrease in its values. That is, depending on the anomalous changes in the geophysical signal, it is possible to make a prediction. Usually, as a result of forecasting, it will be possible to predict such information as the location of the most accumulated minerals, their reserve capacity. Anomalous changes in the electromagnetic and gravitational fields of the earth, abnormal changes in the ionosphere, seismic conditions (noise), various acoustic vibrations can be used as a tree [1].

In recent years, dozens of methods for predicting minerals have been proposed by scientists around the world. One of them is the spline method. Using the spline method, scientists have the opportunity to understand the physical processes that take place underground, to observe them, to build mathematical models of the interdependence of these processes. This is mainly achieved by using two-dimensional cupcake spline functions to achieve a high level of accuracy. The two-dimensional cubic spline function we offer also has a high accuracy range.

2. Modeling of geophysical signals based on the second-order local interpolication spline-function

The application of parabolic local interpolation spline functions to build a mathematical model for the digital processing and recovery of various geophysical signals is currently relevant in solving practical problems.

The $S_n(f; x)$ function under consideration is called the local interpolation spline function when the following conditions are met:

1. $S_n (f; x) \in H_n[x_i, x_{i+1}]$,

2. $S_n (x) \in C^m[a,b]$,

3. $S_n (x_i) = f(x_i)$ i=0,n.

Based on the above conditions, we consider the construction of a local interpolation bicubic spline in the following $D=[a,b]\times[c,d]$ area. We can divide the area under consideration into N equal parts along the OX axis and M along the OY axis $\Delta=\Delta x\times\Delta y$. [1].

$$\Delta_x: \ a = x_0 < x_1 <\ldots< x_N = b, \ \Delta_y: \ c = y_0 < y_1 <\ldots< y_M = d.$$

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where steps h and l are selected as follows

$$h = x_{i+1} - x_i, \; i=0,1,...,N-1; \; l = y_{j+1} - y_j,$$

$$j = 0,1,\ldots,M-1.$$

Let we look at the grid below:

$$\Delta^* = \Delta_x^* \times \Delta_y^*$$

$$\Delta_x^*: \; x_{-1} < x_0 < x_1 < \ldots < x_N < x_{N+1}, \quad \Delta_y^*: \; y_{-1} < y_0 < y_1 < \ldots < y_M < y_{M+1}.$$

Δ_x^* for this case we make the field look $D^* = [a - h, b + h] \times [c - l, d + l]$. Δ^*- At the node points in the grid, the values of the function are known, i.e.:

$$f(x_i, y_j) = f_{ij}, \quad i = -1,0,1, \ldots, N, N + 1;$$

$$j = -1,0,1,\ldots,M, M + 1.$$

Based on the above values of the $f(x,y)$ - function, we construct a local interpolation spline function for the field. To do this, we use the following 16 points.

$$(x_{i-1}, y_{j-1}), \; (x_{i-1}, y_j), \; (x_{i-1}, y_{j+1}), (x_{i-1}, y_{j+2}),$$

$$(x_i, y_{j-1}), \; (x_i, y_j), \; (x_i, y_{j+1}), \; (x_i, y_{j+2}),$$

$$(x_{i+1}, y_{j-1}), \; (x_{i+1}, y_j), \; (x_{i+1}, y_{j+1}), \; (x_{i+1}, y_{j+2}),$$

$$(x_{i+2}, y_{j-1}), \; (x_{i+2}, y_j), \; (x_{i+2}, y_{j+1}), \; (x_{i+2}, y_{j+2}).$$

Our article in Comparative Analysis Spline Methods in Digital Processing of Signals in ASTESJ magazine describes the construction of a one-dimensional local cubic spline using 4 points [2]. Here, too, $Z_j(x_i, j)$, $Z_{j+1}(x_i, j)$ parabolas are constructed using the above points. Here x is fixed, i.e. at $x=x_i$ the local interpolation bi cubic spline-function $S_3(x_i, y)$ has the following form:

$$S_3(x_i, y) = (1 - u)Z_j(x_i, y) + uZ_{j+1}(x_i, y), \qquad (1)$$

here it is

$$Z_j(x_i, y) = -\frac{1}{2}u(1 - u)f_{i,j-1} + (1 - u^2)f_{ij} + \frac{1}{2}u(1 + u)f_{i,j+1}, \tag{2}$$

$$Z_{j+1}(x_i, y) = \frac{1}{2}(1 - u)(2 - u)f_{ij} + u(2 - u)f_{i,j+1} - \frac{1}{2}u(1 - u)f_{i,j+2} \tag{3}$$

$Z_j(x_i, j)$, $Z_{j+1}(x_i, j)$ parabolas are as follows

$$(x_i, y_{j-1}), \; (x_i, y_j), \; (x_i, y_{j+1});$$

$$(x_i, y_j), \; (x_i, y_{j+1}), \; (x_i, y_{j+2}),$$

$$u = \frac{y - y_j}{l}, \; l = y_{j+1} - y_j.$$

passes through node points. Substituting (2) and (3) into (1), we obtain the following after certain reductions:

$$S_3(x_i, y) = -\frac{1}{2}u(1 - u)^2 f_{i,j-1} + \frac{1}{2}(1 - u)(2 +$$

$$2u - 3u^2)f_{ij} + +\frac{1}{2}u(1 + 4u -$$

$$3u^2)f_{i,j+1} - \frac{1}{2}u^2(1 - u)f_{i,j+2},$$

$$j = \overline{0, M - 1}, \; 0 \le u \le 1.$$

Based on the above $x = x_{i-1}; \quad x_{i+1}; \quad x_{i+2}$. In fixed cases, we create the following single-variable spline functions

$$S_3(x_{i-1}, y) = (1 - u)Z_j(x_{i-1}, y) + uZ_{j+1}(x_{i-1}, y), \quad (5)$$

$$S_3(x_{i+1}, y) = (1 - u)Z_j(x_{i+1}, y) + uZ_{j+1}(x_{i+1}, y), \quad (6)$$

$$S_3(x_{i+2}, y) = (1 - u)Z_j(x_{i+2}, y) + uZ_{j+1}(x_{i+2}, y), \quad (7)$$

$S_3(x_{i-1}, y)$, $S_3(x_i, y)$, $S_3(x_{i+1}, y)$ and $S_3(x_{i+2}, y)$ Based on the one-variable cubic spline functions built above, after a certain compression, the following view of the following two-variable interpolation bicubic spline function is formed [3-7]:

$$S_{3,3}(x, y) = -\frac{1}{2}t(1 - t)^2 S_3(x_{i-1}, y) + \frac{1}{2}(1$$

$$-t)S_3(x_i, y) + \frac{1}{2}t(1 + 4t - 3t^2)S_3(x_{i+1}, y) -$$

$$\frac{1}{2}t^2(1 - t)S_3(x_{i+2}, y),$$

$$j = \overline{0, M - 1}, 0 \le u \le 1, t = \frac{x - x_i}{h}, u = \frac{y - y_j}{l}, h = x_{i+1} - x_i, l = y_{j+1} - y_j.$$

$S_3(x_{i-1}, y)$, $S_3(x_i, y)$, $S_3(x_i+1, y)$ and $S_3(x_{i+2}, y)$ are generated by putting the values of the bicubic spline functions of a variable built above (8) [8-12].

$$S_{3,3}(x, y) = -\frac{1}{2}t(1 - t^2)\left[(1 - u)Z_j(x_{i-1}, y) + uZ_{j+1}(x_{i-1}, y)\right]$$

$$+\frac{1}{2}(1 - t)(2 + 2t - 3t^2)\left[(1 - u)Z_j(x_i, y) + uZ_{j+1}(x_i, y)\right]$$

$$+\frac{1}{2}t(1 + 4t - 3t^2)\left[(1 - u)Z_j(x_{i+1}, y) + uZ_{j+1}(x_{i+1}, y)\right]$$

$$-\frac{1}{2}t^2(1 - t)\left[(1 - u)Z_j(x_{i+2}, y) + uZ_{j+1}(x_{i+2}, y)\right]. \tag{8}$$

Here it is

$i = \overline{0, N-1}$, $j = \overline{0, M-1}$, $0 \le t \le 1$, $0 \le u \le 1$, $t = \frac{x - x_i}{h}$,

$u = \frac{y - y_j}{l}$, $h = x_{i+1} - x_i$, $l = y_{j+1} - y_j$.

appears after a certain contraction (9).

$$S_{3,3}(x, y) =$$

$$\varphi_1(t)\big[\varphi_1(u)f_{i-1,j-1} + \varphi_2(u)f_{i-1,j} + \varphi_3(u)f_{i-1,j+1} + \varphi_4(u)f_{i-1,j+2}\big]$$

$$+\varphi_2(t)\big[\varphi_1(u)f_{i,j-1} + \varphi_2(u)f_{i,j} + \varphi_3(u)f_{i,j+1} + \varphi_4(u)f_{i,j+2}\big]$$

$$+\varphi_3(t)\big[\varphi_1(u)f_{i+1,j-1} + \varphi_2(u)f_{i+1,j} + \varphi_3(u)f_{i+1,j+1} + \varphi_4(u)f_{i+1,j+2}\big]$$

$$+\varphi_4(t)\big[\varphi_1(u)f_{i+2,j-1} + \varphi_2(u)f_{i+2,j} + \varphi_3(u)f_{i+2,j+1} + \varphi_4(u)f_{i+2,j+2}\big] \tag{9}$$

here it is

$$\varphi_1(t) = -\frac{1}{2}t(1-t)^2,$$

$$\varphi_2(t) = \frac{1}{2}(1-t)(2 + 2t - 3t^2),$$

$$\varphi_3(t) = \frac{1}{2}t(1 + 4t - 3t^2),$$

$$\varphi_4(t) = -\frac{1}{2}t^2(1-t)$$

$$t = \frac{x - x_i}{h}$$

$$\varphi_1(u) = -\frac{1}{2}u(1-u)^2,$$

$$\varphi_2(u) = \frac{1}{2}(1-u)(2 + 2u - 3u^2),$$

$$\varphi_3(u) = \frac{1}{2}u(1 + 4u - 3u^2),$$

$$\varphi_4(u) = -\frac{1}{2}u^2(1-u)$$

$$u = \frac{y - y_j}{l}$$

(9) can be called a local interpolation bicubic spline function.

3. Digital processing of geophysical signals based on developed algorithms

The proposed model considered the recovery of the signal obtained by aeromagnetic reconnaissance to determine the location of underground mineral resources, which is considered to be our current object (Table 1). Based on the above sequence, a local interpolation bicubic spline construction program was developed in the Matlab program environment and used in signal processing. The algorithm of this program is shown in Figure 3. We have shown the superiority of our model by comparing our developed model with the bicubic spline function S_{bsa} (10) model, which is based on the existing basic functions. Since it is not possible to cite all the known signals, we present the result in the following figures using the appearance of the signals in Table 1 (Figures 1-2). As can be seen from these pictures, it is clear from the $S_{3,3}$ and S_{bsa} spline functions that the degree of approximation of the $S_{3,3}$ we offer is high [13-16].

$$S_{bsa}(x, y) = \omega_1(p)\big[\omega_1(q)f_{i-1,j-1} + \omega_2(q)f_{i-1,j} + \omega_3(q)f_{i-1,j+1} + \omega(q)f_{i-1,j+2}\big]$$

$$+\omega_2(p)\big[\omega_1(q)f_{i,j-1} + \omega_2(q)f_{i,j} + \omega_3(q)f_{i,j+1} + \omega_4(q)f_{i,j+2}\big]$$

$$+\omega_3(p)\big[\omega(q)f_{i+1,j-1} + \omega_2(q)f_{i+1,j} + \omega_3(q)f_{i+1,j+1} + \omega_4(q)f_{i+1,j+2}\big]$$

$$+\omega_4(p)\big[\varphi_1(q)f_{i+2,j-1} + \omega_2(q)f_{i+2,j} + \omega_3(q)f_{i+2,j+1} + \omega_4(q)f_{i+2,j+2}\big]. \tag{10}$$

here it is

$$\omega_1(p) = (1-p)(1-p)(1-p),$$

$$\omega_2(p) = (3p^3 - 6p^2 + 4),$$

$$\omega_3(p) = (1 + 3p + 3p^2 - 3p^3),$$

$$\omega_4(p) = p^3$$

$$\omega_1(q) = (1-q)(1-q)(1-q),$$

$$\omega_2(q) = (3q^3 - 6q^2 + 4),$$

$$\omega_3(q) = (1 + 3q + 3q^2 - 3q^3),$$
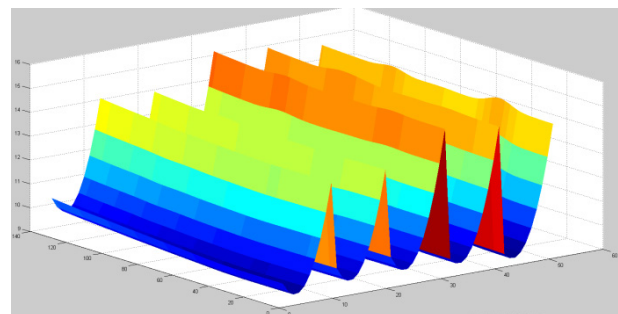
$$\omega_4(q) = q^3$$

Figure 1: Graphical representation of the result obtained in the S_{bsa} function of the geophysical field

We used signals as real experimental data during our research. The obtained results are used to determine the location, size and amount of reserves of mineral resources by means of aerial magnetic intelligence. Of course, signal charts here use signals in the low frequency range from 10 to 50 kHz. Also, the distances between the nodes in the area under consideration must be equal [17-19].
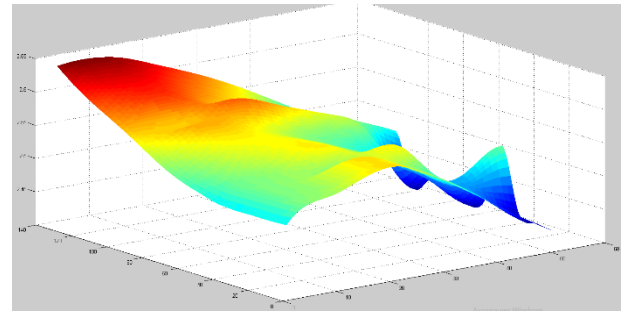

Figure 2: Graphical representation of the result obtained in the $S_{3,3}$ function of the geophysical field
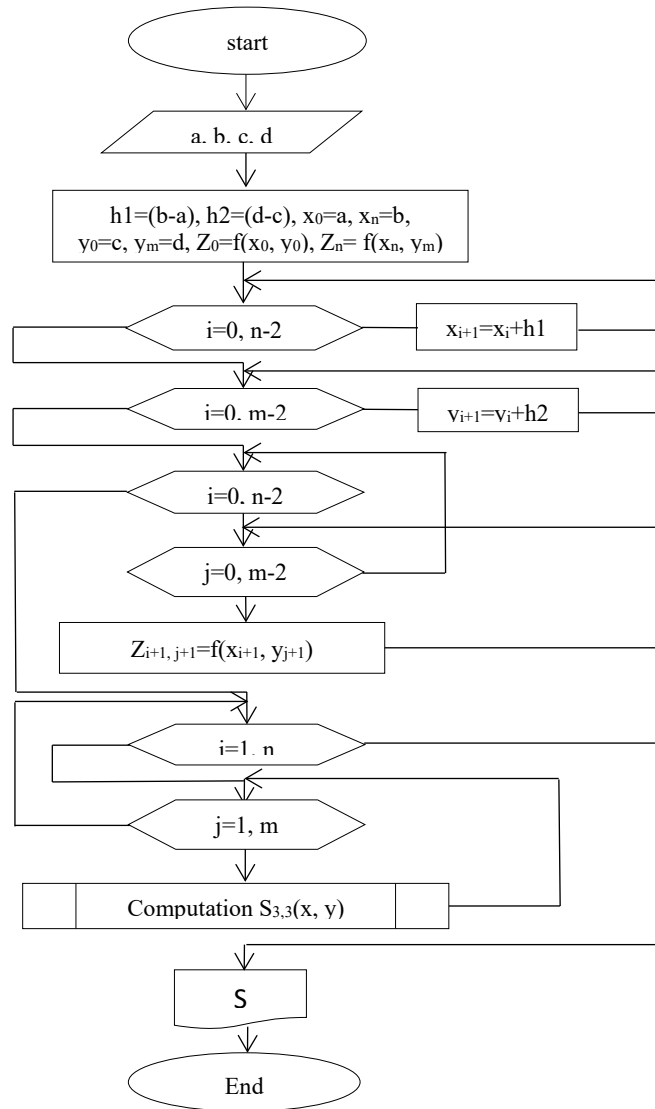

Figure 3: Block diagram of a two-dimensional bicubic spline-function construction program.

Table 1: Initial values of the geophysical signal

xi, yi	1	2	3	4	5	6	7
1	2.511567	2.546819	2.570055	2.534253	2.479736	2.428968	2.396564
2	2.510239	2.542991	2.568838	2.534879	2.484192	2.432505	2.399471
3	2.510149	2.541944	2.568157	2.536136	2.489958	2.437171	2.40337
4	2.509842	2.53978	2.53978	2.56434	2.489958	2.533676	2.488477

| 5 | 2.512262 | 2.540757 | 2.564102 | 2.534735 | 2.493134 | 2.440931 | 2.407022 |
6	2.520041	2.549419	2.564856	2.536827	2.50416	2.428968	2.447254
7	2.528682	2.556995	2.562458	2.535142	2.493763	2.44869	2.428968
8	2.541257	2.565523	2.560248	2.533722	2.503736	2.450393	2.416823
9	2.557538	2.581372	2.56551	2.53978	2.53651	2.428968	2.503185
10	2.57455	2.57455	2.597675	2.571599	2.541111	2.505716	2.460124
11	2.589951	2.605743	2.578684	2.546592	2.508904	2.464554	2.431107
12	2.605048	2.615953	2.588514	2.555813	2.516062	2.471976	2.438394
13	2.622756	2.628817	2.600686	2.566327	2.524289	2.47895	2.428968
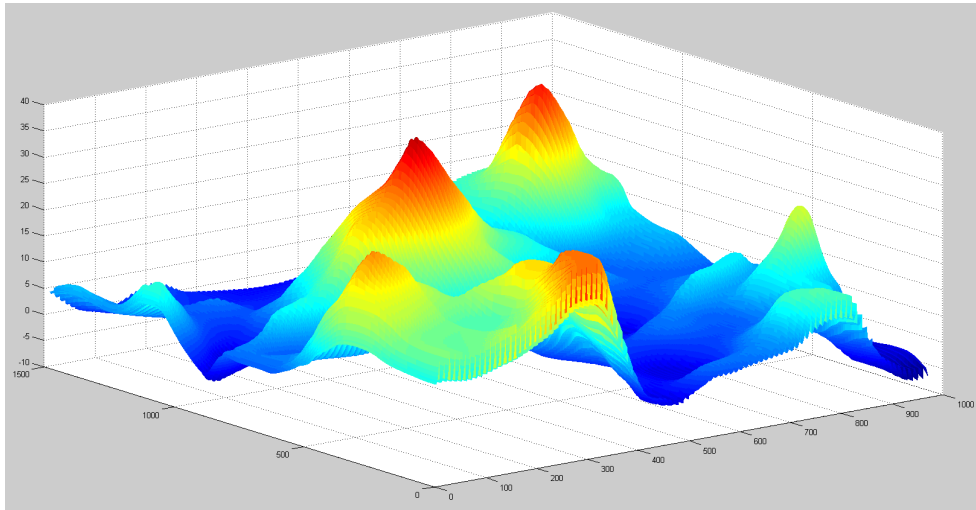
Figure 4: Graphic representation of the result obtained by the S_{bsa} function with the geophysical field (0.1x0.1) step.
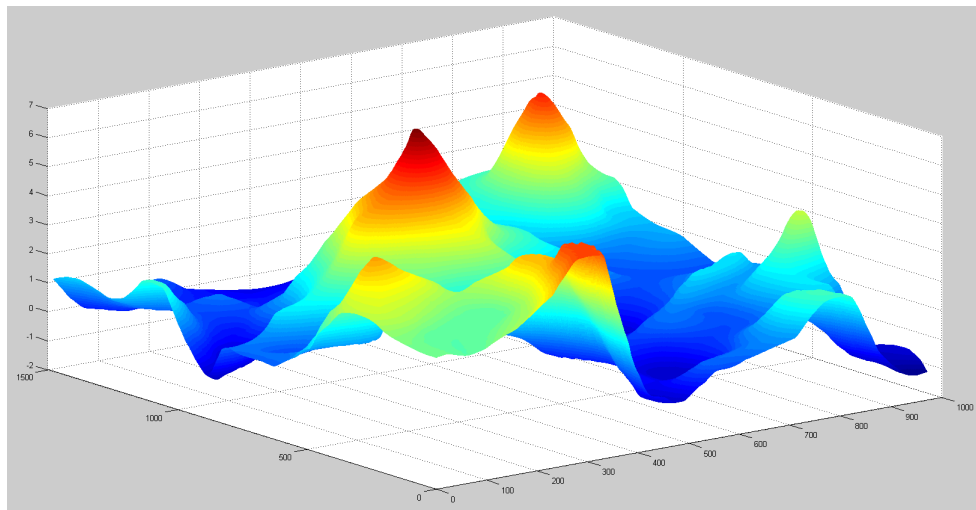

Figure 5: Graphic representation of the result obtained by the $S_{3,3}$ function with the geophysical field (0.1x0.1) step.

Table 2: Results in error estimation in geophysical signal recovery.

x_i	y_i	$f(x,y)$	$S_{3,3}$	S_{bsa}	$|S_{3,3} - f(x,y)|$	$|S_{bsa} - f(x,y)|$
1	1	2,5116	2.5116	9,9534	0	7,4418
1.1	1	2,5115	2.5114	9.9526	0,0001	7,4412
1.2	1	2,5113	2.5114	9.9517	0,0001	7,4404
1.3	1	2,5111	2.5111	9.9510	0,0001	7,4399
1.4	1	2,5108	2.5110	9.9502	0,0002	7,4394
1.5	1	2,5105	2.5108	9.9496	0,0005	7,4391
1.6	1	2,5104	2.5107	9.9491	0,0003	7,4387
1.7	1	2,5096	2.5105	9.9486	0,001	7,439
1.8	1	2,509	2.5104	9.9482	0,0014	7,4392

1.9	1	2,5084	2.5103	9.9479	0,0021	7,4395
2	2	2.5102	2.5102	9.9478	0	7,4376
2.1	2	2,5101	2.5102	9.9477	0,0001	7,4376
2.2	2	2,5102	2.5101	9.9478	0,0001	7,4376
2.3	2	2,5102	2.5101	9.9479	0,0001	7,4377
2.4	2	2,5102	2.5101	9.9481	0,0001	7,4379
2.5	2	2,5102	2.5101	9.9484	0,0001	7,4382
2.6	2	2,5101	2.5101	9.9487	0	7,4386
2.7	2	2,5101	2.5102	9.9491	0,0001	7,439
2.8	2	2,5101	2.5102	9.9494	0,0001	7,4393
2.9	2	2,5101	2.5102	9.9499	0,0001	7,4398
3	3	2.5101	2.5101	9.9503	0	7,4402
Max					0,0021	7,4418

From the graphic images and tables it can be seen that the local interpolation bicubic spline function has a high degree of accuracy in the digital processing of signals.

4. Conclusion

The paper, a cubic spline built using a local base spline for digital signal processing and the local interpolation bicubic spline models we offer were selected. Construction details of the models are given. Two-dimensional local interpolation bicubic spline models provide high accuracy in digital processing of signals. As an example, the initial values of the geophysical signal given in Table 1 were numerically processed and the error results were obtained (Table 2). Based on the selected models, the results of geophysical signal recovery are presented graphically (Figures 1,2,3 and 4). Based on error results and graphs, it is effective to apply the two-dimensional local bicubic spline model in areas of geophysics and high accuracy.

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