The Impact Assessment of the Errors in Determining the Mass and Zero Lift-Drag Coefficient on the Aircraft's Performance Data

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A R T I C L E   I N F O

A B S T R A C T

The adoption of technical decisions in the development of a new aircraft is carried out under the condition of insufficient information on the mass - dimensional and aerodynamic characteristics that will be achieved during the project. Insufficient information on the input parameters (weight and aerodynamic characteristics) leads to uncertainties in the values of the flying qualities and performances. When creating a new aircraft, the chief designer and specialists of the design bureaus establish certain reserves (limits) for mass and aerodynamic characteristics. The allocation of reserves for the characteristics of aircraft elements is carried out, as a rule, on the basis of experience gained from previous projects, and to a certain extent subjective. At the same time, it is not possible to quantify the risk of non-fulfillment of the tactical and technical task in terms of performance characteristics. Knowing the contribution of the uncertainties of the input parameters (for the values of which a margin is allocated) to the uncertainty of the final values of the flight characteristics will allow the person who makes the decision to reasonably establish reserves in the design of the aircraft and take a more reasonable risk. In this paper, the authors describe an approach that allows for a quantitative assessment of the impact assessment of the errors in determining the aircraft's characteristics on the tactical and technical task in terms of the aircraft's performance data. Using the example of the mass and zero lift-drag coefficient, analytical dependencies are derived that allow a quantitative assessment of the effect. To verify the obtained analytical dependencies, the calculation of weight coefficients was made for several aviation complexes of operational-tactical aviation. The analysis of the sensitivity of the aircraft's performance data to the parameters under consideration is carried out.

1. Introduction

When developing new aircraft, the most important decisions that determine the future fate of the project are made at the initial stages of work in conditions of uncertainty about the impact of these decisions on the performance of the – tactical and technical task (TTT). Mistakes in the early stages of design tend to cost more money and time for their elimination at the subsequent stages of the aircraft, and sometimes on the correctness of decisions depends on the feasibility of the project as a whole.

The uncertainty of the performance of the tactical and technical assignment in terms of flight characteristics at the stage of internal and external design is associated with many factors. For example,

a delay in the receipt from related enterprises of reliable data on the characteristics of aircraft systems (such as engine, target equipment, etc.), insufficient knowledge of some physical laws, which is most pronounced when new technical solutions are introduced, and others.

When predicting the level of performance characteristics of a new aircraft, they rely on indicators of its technical perfection, which will be achieved as a result of the project. These include the relative mass of the structure and systems, specific engine thrust, specific fuel consumption, aerodynamic quality, and others. With the development of aviation, these indicators are constantly improving.

The choice of a reasonable level of technical excellence, and therefore TTT in the design of a new aircraft, is an important task. Excessive overestimation of the requirements for its weight and

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aerodynamic characteristics can lead not only to non-fulfillment of the promising requirements for flight performance, but also deterioration of the flight characteristics of the designed aircraft relative to the current level already achieved due to the mismatch of the characteristics of its elements. At the same time, the use of previously developed technical solutions and technologies, in which there is no risk, does not make it possible to ensure the required increase in flight performance and, as a consequence, efficiency in the development of new generation aircraft. Therefore, at the initial design stage, when applying certain technical solutions, it is necessary to have a clear idea of the real possibilities of performing the performance characteristics specified in the TTT.

In accordance with the existing design methodology, when creating a new aircraft, the chief designer and specialists of the design bureaus establish certain reserves (limits) for mass and aerodynamic characteristics. The reason for the allocation of such reserves is that at the initial design stage there is no exact idea of the final value of mass and aerodynamic characteristics, which will be obtained upon completion of the development stage. Each aircraft has its own targets. Therefore, the characteristic for which the reserve is allocated depends on the specific type of aircraft. For example, the characteristics of a commercially available engine are known values. If the power plant is at the development stage, there is a risk of non-implementation of the TTT of the engine, which may affect the non-implementation of the TTT of the aircraft. The use of insufficiently mastered materials or technologies in the aircraft structure can lead to a significant deviation in the mass characteristics of the airframe units, etc. Therefore, the identification of the main elements that introduce uncertainty in the level of aircraft performance should be made in each case specifically.

The allocation of reserves for the characteristics of aircraft elements is carried out, as a rule, on the basis of experience gained from previous projects, and to a certain extent subjective. At the same time, it is not possible to quantify the risk of non-fulfillment of TTT in terms of performance characteristics. Knowing the contribution of the uncertainties of the input parameters (for the values of which a margin is allocated) to the uncertainty of the final values of the flight characteristics will allow the person who makes the decision to reasonably establish reserves in the design of the aircraft and take a more reasonable risk.

Approaches to the formation of the appearance of an aircraft with inaccurate determination of the initial data are reflected in many scientific works, for example [1-10]. The relevance of this issue is also confirmed by scientific works [11-15].

To assess the technical solutions adopted in the design, it is necessary to associate particular changes in various parameters (weight, aerodynamic characteristics) with a change in the aircraft flight characteristics.

2. Materials and Methods

Let's introduce the concept of the influence coefficient (IC) – a dimensionless coefficient showing the ratio of the change in function (*f*) to the change in input parameters (x_i) in the relative form:

$$K_{x_{i\_f}} = \frac{\Delta \hat{f}}{\Delta \hat{x}_i} \tag{1}$$

where $\hat{f} = \frac{\Delta f}{f_1}$, $\Delta \hat{x}_i = \frac{\Delta x_i}{x_{i_1}}$, Δf and Δx_i – uncertainty of the function and the input parameters, f_l and x_{i_1} - expected value of the function and the input parameters.

In the absence of a relationship between the initial parameters, ICs are the coefficients of linear decomposition of the inaccuracy of determining the function from the errors in determining the parameters (in the relative form):

$$\frac{\Delta f}{f_{expect}} = \sum_{i=1}^{n} K_{x_{i\_f}} \frac{\Delta x_i}{x_{i_{expect}}} \tag{2}$$

The numerical value of the IC is equal to the percentage change of flying qualities and performances (FQP) when the parameter changes to 1%.

This paper assesses the influence of such parameters as mass (*m*) and zero lift-drag coefficient (Cx_0) on:
- maximum flying speed - V_{max};
- service ceiling - H_{max};
- specific excess power (max) - V_y^*;
- sustained load factor - n_y;
- acceleration time - t_p.

For a comprehensive study of the considered IC, the research is carried out in two directions:
1. Analytically, by deriving IC from the formula dependencies of the FQP;
2. Numerical calculations of the IC by calculating the changes in FQP with varying mass and zero lift-drag coefficient.

The analytical method makes it possible to establish the dependence of IC on specific aircraft parameters. These dependencies are derived from the flight dynamics formulas and can be applied to absolutely any aircraft with a similar flight principle. However, to represent the dependencies in a relatively simple and easy-to-use form, it is necessary to introduce some assumptions that can be used to perform the required mathematical transformations. The formulas obtained require verification, since one and the same introduced assumption can give different errors in magnitude, depending on the type of aircraft and its purpose.

The verification of the derived formulas for IC can be carried out by the calculation method. The computational method, in contrast to the analytical one, allows one to obtain the exact value of IC (without assumptions), but does not allow one to establish the dependence of IC on the aircraft parameters. The IC is calculated using the FQP calculation program detailed in section 2.

Thus, comparison of the analytical findings with numerical calculations allows to prove the accuracy of the first.

3. Calculation of the influence coefficient by analytically method

For various parameters, the degree of their uncertainty can differ significantly depending on the level of elaboration of certain elements of the aircraft. The accuracy of calculating the mass of

aircraft at the stage of the preliminary project is 6-10%, outline design 3-5%, working draft 1-2% [5].

The influence coefficient, as described above, is indicated $K_{x_{i\_f}}$. According to the IC mass for sustained load factor is indicated $K_{m\_n_y}$, IC C_{x_0} to specific excess power - $K_{C_{x_0}\_V_y^*}$, etc. We use the following notation:

$$\Delta \hat{m} = \frac{m_2 - m_1}{m_1} \quad \Delta \widehat{Cx_0} = \frac{Cx_{0_2} - Cx_{0_1}}{Cx_{0_1}}$$ – relative changes in mass and zero lift-drag coefficient.

3.1. The influence of parameters on maximum flying speed

The maximum flying speed can be expressed from the equality of thrust and drag:

$$V_{max} = \sqrt{\frac{2P_{max}}{(C_{x_0} + C_{x_i})\rho S}} \tag{3}$$

where C_{x_i} – inductive reactance coefficient, P_{max} – thrust at which maximum speed is reached, S – wing area, ρ – air density.

The inductive component of resistance when flying at maximum speed is small. In this case $C_{xi} \approx 0$ and the maximum flight speed can be calculated by the formula:

$$V_{max} = \sqrt{\frac{2P_{max}}{C_{x_0}\rho S}} \tag{4}$$

Let's get the formula for $K_{Cx_0 - V_{max}}$:

$$K_{Cx_0 - V_{max}} = \frac{\frac{V_{max2}}{V_{max1}} - 1}{\frac{Cx_{0_2}}{Cx_{0_1}} - 1} = \frac{\sqrt{\frac{Cx_{0_1}}{Cx_{0_2}}} * \sqrt{\frac{P_2}{P_1}} - 1}{\frac{Cx_{0_2}}{Cx_{0_1}} - 1}$$

Let $\tilde{c} = \frac{Cx_{0_2}}{Cx_{0_1}}$, and the coefficient $\tilde{p} = \frac{P_2}{P_1}$ will show the characteristics' flow of the altitude-speed performance (ASP) (with a change in V_{max}, the engine thrust will change). Then the influence coefficient Cx_0 to V_{max} will be equal to:

$$K_{Cx_0 - V_{max}} = -\frac{1 + \sqrt{\frac{\tilde{p}}{\tilde{c}}}}{\tilde{c} - 1} \tag{5}$$

The coefficient $\tilde{p}$ corresponds to a specific aircraft because its value is influenced by both the engine's ASP and the intake losses ($\Delta \bar{p}_{вх}$) and nozzle ($\Delta \bar{p}_c$):

$$\tilde{p} = f(\Delta \bar{p}_{вх}, \Delta \bar{p}_c, \tilde{c}) \tag{6}$$

The maximum flying speed is not always determined by the required and available thrusts. Limitations may include temperature, structural strength, stability and control of the aircraft.

As described above, it may be concluded that the biding of the coefficient $K_{Cx_0 - V_{max}}$ to the parameters of the aircraft is impractical, because it will not give a true picture of understanding the impact of the change Cx_0 to V_{max}.

3.2. The influence of parameters on service ceiling

The effect changes on the ceiling can be estimated by the formula [16]:

$$\Delta H_{max} = -6.3 \Delta m \ [km] \tag{7}$$

In this case, IC will be equal to:

$$K_{m\_H_{max}} = \frac{\frac{H_{max_2} - H_{max_1}}{H_{max_1}}}{\frac{m_2 - m_1}{m_1}} = \frac{\frac{-6.3 \Delta \hat{m}}{H_{max_1}}}{\Delta \hat{m}} = \frac{-6.3}{H_{max_1}}$$

$$\boldsymbol{K_{m\_H_{max}} = -\frac{6.3}{H_{max}}} \tag{8}$$

The value of the coefficient is related to the initial ceiling of the aircraft and does not depend on the percentage change in mass (Table 1).

Table 1: The dependence of $K_{m\_H_{max}}$ on the service ceiling.

Initial ceiling H_{max}, km	$K_{m\_H_{max}}$	$\Delta \hat{m}$	ΔH_{max}, %	ΔH_{max}, m
20	-0.315	0.01 (1%)	0.315	63
		0.05 (5%)	1.575	315
18	-0.35	0.01 (1%)	0.315	63
		0.05 (5%)	1.575	315
16	-0.394	0.01 (1%)	0.315	63

In order not to become attached to the initial ceiling, it makes sense to speak not about a relative change in H_{max}, but about the absolute. Therefore, with an increase in mass by 5% the ceiling decreases by 315 meters, 10% - 630 meters. An increase in the mass of 1% leads to a decrease in the ceiling of about 63 meters.

3.3. The influence of parameters on specific express power

In order not to become attached to the initial ceiling, it makes sense to speak not about a relative change in H_{max}, but about the absolute. Therefore, with an increase in mass by 5% the ceiling decreases by 315 meters, 10% - 630 meters. An increase in the mass of 1% leads to a decrease in the ceiling of about 63 meters. We write the specific excess power as follows [16]:

$$V_y^* = \frac{P - X}{G} V \tag{9}$$

where P – engine thrust, X – aircraft drag, G – aircraft weight, V – flight speed.

We assume that the flying speed at which the specific excess power is calculated is constant ($V_1 \approx V_2$), then $K_{m\_V_y^*}$ will be equal to:

$$K_{m\_V_y^*} = \frac{\frac{V_{y_2}^*}{V_{y_1}^*}-1}{\frac{m_2}{m_1}-1} = \frac{\frac{m_1(P-X_2)}{m_2(P-X_1)}-1}{\frac{m_2}{m_1}-1} = \frac{\frac{m_1\left(\frac{P-X_1-\Delta X_i}{P-X_1}\right)}{m_2}-1}{\frac{m_2}{m_1}-1} =$$

$$-\frac{1-\frac{m_1}{m_2}\left(1-\frac{\Delta X_i}{P-X_1}\right)}{\frac{m_2}{m_1}-1} = \langle t = \frac{m_2}{m_1}; K_{X_i} = 1-\frac{\Delta X_i}{P-X_1}\rangle = -\frac{1-\frac{K_{X_i}}{t}}{t-1}$$

$$\boxed{K_{m\_V_y^*} = -\frac{1+\frac{K_{X_i}}{t}}{t-1}} \tag{10}$$

where $t = \frac{m_2}{m_1}; K_{X_i} = 1-\frac{\Delta X_i}{P-X_1}, \Delta X_i$ - increase in inductive component due to mass increase, $\Delta P = P - X$ – abundance of power.

Note that the higher ΔP, the smaller contribution to the value of the IC makes ΔX_i. With small changes in mass (in this case $\Delta X_i \to 0$) and a relativity high value of ΔP we can assume that $K_{m\_V_y^*} \approx -\frac{m_1}{m_2}$. Then at $\Delta m=10\%$ $K_{m\_V_y^*} \approx -0.91$, and the coefficient value without the assumptions described above will be even less.

We similarly take out $K_{C_{x_0}\_V_y^*}$:

$$K_{C_{x_0}\_V_y^*} = \frac{\frac{V_{y_2}^*}{V_{y_1}^*}-1}{\frac{Cx_{0_2}}{Cx_{0_1}}-1} = \frac{\left(\frac{P-X_2}{P-X_1}\right)-1}{\frac{Cx_{0_2}-Cx_{0_1}}{Cx_{0_1}}} = \frac{\left(\frac{P-X_1-\Delta X_0}{P-X_1}\right)-1}{\frac{Cx_{0_2}-Cx_{0_1}}{Cx_{0_1}}} = -\frac{1-\frac{\Delta X_0}{P-X_1}-1}{\frac{Cx_{0_2}-Cx_{0_1}}{Cx_{0_1}}} =$$

$$\langle \tilde{c} = \frac{Cx_{0_2}}{Cx_{0_1}}\rangle = -\frac{\Delta X_0 Cx_{0_1}}{\Delta P(Cx_{0_2}-Cx_{0_1})} = -\frac{(Cx_{0_2}-Cx_{0_1})\frac{\rho V^2 S}{2}Cx_{0_1}}{\Delta P(Cx_{0_2}-Cx_{0_1})} = -\frac{X_0}{\Delta P}$$

$$\boxed{K_{C_{x_0}\_V_y^*} = -\frac{X_0}{\Delta P}} \tag{11}$$

If we neglect the inductive component of the resistance, then the formula takes the form:

$$\boxed{K_{C_{x_0}\_V_y^*} = -\frac{1}{\frac{P}{X_0}-1}} \tag{12}$$

The value of the coefficient does not depend on the percentage change C_{x_0} and is associated with a specific aircraft.

The max value of the specific excess power for fighters is achieved as a rule at M=0.8-0.9 near the ground. We calculate the IC value according to the formula (11) and (12) for several aircraft. At M=0.85 ram airflow is q=51250 H/m², the IC excluding and taking into account the inductance is presented in Table 2.

Table 2: The values $K_{C_{x_0}\_V_y^*}$ are calculated by the formula (4) and (5)

Aircraft	M	Cx_0	Cx_i	S, m²	P, H	$K_{C_{x_0}\_V_y^*}$ excluding Cx_i	$K_{C_{x_0}\_V_y^*}$ taking into account Cx_i
Mig-29	0.9	0.025	0.0005	38	200000	**-0.398**	**-0.401**
Su-24 (69°)	0.8	0.0263	0.0029	51	230000	**-0.367**	**-0.383**
Mig-23 (72°)	0.9	0.0225	0.0026	34.2	150000	**-0.438**	**-0.460**

From Table 2 it is seen that the inductive component of the resistance makes an insignificant contribution to the value of the IC. When Cx_0 is changed by 10% the differences in the scatter of V_y^* excluding and taking into account Cx_i are about 0.2%, which can be considered a negligible value at the preliminary design stage. Therefore, to simplify, you can use the formula (5).

3.4. The influence of parameters on sustained load factor

The sustained load factor can be expressed through the formula [16]:

$$n_y = \sqrt{\frac{(P-X_0)qS}{A(mg)^2}} \tag{13}$$

where P – engine thrust, X_0 – non-inductive resistance, q – velocity head, A – drag-due-to-lift factor, g – acceleration of gravity, then:

$$K_{m\_n_{y\,уст}} = \frac{\frac{n_{y_2}}{n_{y_1}}-1}{\frac{m_2}{m_1}-1} = \frac{\frac{m_1}{m_2}-1}{\frac{m_2}{m_1}-1} = \langle t = \frac{m_2}{m_1}\rangle = \frac{\frac{1}{t}-1}{t-1} = \frac{1-t}{t(t-1)} = -\frac{1}{t}$$

$$\boxed{K_{m\_n_y} = -\frac{m_1}{m_2}} \tag{14}$$

The value of the coefficient depends on the percentage change in mass (Table 3).

Table 3: Dependence of $K_{m\_n_y}$ on Δm

Mass change Δm, %	$K_{m\_n_y}$	Change of the sustained load factor Δn_y, %
5	**-0.952**	*4.8*
10	**-0.909**	*9.1*
15	**-0.869**	*13*

Similarly, we get $K_{Cx_0\_n_y}$:

$$K_{Cx_0\_n_{y\,уст}} = \frac{\frac{n_{y_2}}{n_{y_1}}-1}{\frac{Cx_{0_2}}{Cx_{0_1}}-1} = \frac{\sqrt{\left(\frac{P-X_2}{P-X_1}\right)}-1}{\frac{Cx_{0_2}-Cx_{0_1}}{Cx_{0_1}}} = \frac{\sqrt{\left(\frac{P-X_1-\Delta X_0}{P-X_1}\right)}-1}{\frac{Cx_{0_2}-Cx_{0_1}}{Cx_{0_1}}} = \langle Cx_{0_2} -$$

$$Cx_{0_1} > 0\rangle = \sqrt{\left(\frac{P-X_1-\Delta X_0}{P-X_1}\right)\frac{(Cx_{0_1})^2}{(Cx_{0_2}-Cx_{0_1})^2}} - \frac{1}{\Delta \widehat{Cx_0}} =$$

$$\sqrt{\left(1-\frac{\Delta X_0}{P-X_1}\right)\frac{Cx_{0_1}^2}{(Cx_{0_2}-Cx_{0_1})^2}} - \frac{1}{\Delta \widehat{Cx_0}} =$$

$$\sqrt{\frac{Cx_{0_1}^2}{(Cx_{0_2}-Cx_{0_1})^2} - \frac{qS(Cx_{0_2}-Cx_{0_1})}{qS\left(\frac{P}{Sq}-Cx_{0_1}\right)}\frac{Cx_{0_1}^2}{(Cx_{0_2}-Cx_{0_1})^2}} - \frac{1}{\Delta \widehat{Cx_0}} =$$

$$\sqrt{\frac{1}{(\Delta\widehat{Cx}_0)^2} - \frac{Cx_{0_1}^2}{(\frac{P}{Sq}-Cx_{0_1})(Cx_{0_2}-Cx_{0_1})}} - \frac{1}{\Delta\widehat{Cx}_0} =$$

$$\sqrt{\frac{1}{(\Delta\widehat{Cx}_0)^2} - \frac{Cx_{0_1}}{(\frac{P}{Sq}-Cx_{0_1})(\Delta\widehat{Cx}_0)}} - \frac{1}{\Delta\widehat{Cx}_0} =$$

$$\sqrt{\frac{1}{(\Delta\widehat{Cx}_0)^2} - \frac{Cx_{0_1}\Delta\widehat{Cx}_0}{(\frac{P}{Sq}-Cx_{0_1})}\frac{1}{(\Delta\widehat{Cx}_0)^2}} - \frac{1}{\Delta\widehat{Cx}_0} = \frac{1}{\Delta\widehat{Cx}_0}\sqrt{1 - \frac{Cx_{0_1}\Delta\widehat{Cx}_0}{\frac{P}{Sq}-Cx_{0_1}}} -$$

$$\frac{1}{\Delta\widehat{Cx}_0} = \frac{1}{\Delta\widehat{Cx}_0}\left(\sqrt{1 - \frac{Cx_{0_1}\Delta\widehat{Cx}_0}{\frac{P}{Sq}-Cx_{0_1}}} - 1\right)$$

$$K_{Cx_0\_n_y} = \frac{1}{\Delta\widehat{Cx}_0}\left(\sqrt{1 - \frac{Cx_{0_1}\Delta\widehat{Cx}_0}{\frac{P}{Sq}-Cx_{0_1}}} - 1\right) \qquad (14)$$

The coefficient value slightly depends on the percentage change Cx_0 (differences – in thousandths). When Cx_0 changes by 10%, the formula takes the form:

$$\boldsymbol{K_{Cx_0\_n_y} = 10\left(\sqrt{1 - 0.1\frac{Cx_{0_1}}{\frac{P}{Sq}-Cx_{0_1}}} - 1\right)} \qquad (15)$$

For example, we calculate the IC for several AK. At M=0.8 at a higher H=1000m ram-air flow is equal q=41200 H/m², (Table 4).

Table 4: The values $K_{Cx_0\_n_y}$

Aircraft	Cx_0	S, m²	P (H=1000, M=0.8), H	$K_{Cx_0\_n_y}$
Mig-29	0.024	38	170000	**-0.16**
Su-24 (69°)	0.026	51	206000	**-0.21**
Mig-23 (72°)	0.022	34.16	130000	**-0.18**

3.5. The influence of parameters on acceleration time

Acceleration time in horizontal flight in within speed range is [17]:

$$t_p = \frac{1}{g}\int_{V_H}^{V_K}\frac{dV}{n_{xa}} \approx \frac{V_K - V_H}{g*(n_{xa})_{cp}} \qquad (16)$$

Where V_H and V_K – initial and final speed, $(n_{xa})_{cp}$ – averaged longitudinal g load for within speed range.

Let's get the formula for $K_{m\_t_p}$:

$$\Delta(n_{xa})_{cp} = (n_{xa})_{cp2} - (n_{xa})_{cp1} = \frac{(P_{cp}-X_{2cp})}{G_2} - \frac{(P_{cp}-X_{1cp})}{G_1} =$$

$$\frac{\frac{m_1}{m_2}P_{cp}-\frac{m_1}{m_2}X_{1cp}-\frac{m_1}{m_2}\Delta X - P_{cp}+X_{1cp}}{G_1} = -\frac{P_{cp}\left(1-\frac{m_1}{m_2}\right)-X_{1cp}\left(1-\frac{m_1}{m_2}\right)+\Delta X\frac{m_1}{m_2}}{G_1} =$$

$$-\frac{(P_{cp}-X_{1cp})\left(1-\frac{m_1}{m_2}\right)+\Delta X\frac{m_1}{m_2}}{G_1} , \text{ then:}$$

$$K_{m\_t_p} = \frac{\frac{t_{p2}}{t_{p1}}-1}{\frac{m_2}{m_1}-1} = \left(\frac{t_{p2}}{t_{p1}}-1\right)\frac{1}{\Delta\widehat{m}} = \left(\frac{(n_{xa})_{cp1}-(n_{xa})_{cp2}}{(n_{xa})_{cp2}}\right)\frac{1}{\Delta\widehat{m}} =$$

$$\left(\frac{1}{\frac{(n_{xa})_{cp2}}{-\Delta(n_{xa})_{cp}}}\right)\frac{1}{\Delta\widehat{m}} = \left(\frac{1}{-1-\frac{\Delta(n_{xa})_{cp}}{(n_{xa})_{cp1}}}\right)\frac{1}{\Delta\widehat{m}} =$$

$$\left(\frac{1}{\frac{1}{\frac{(P_{cp}-X_{1cp})\left(1-\frac{m_1}{m_2}\right)G_1}{(P_{cp}-X_1)G_1}+\frac{\Delta X*m_1*G_1}{m_2(P_{cp}-X_1)G_1}}-1}\right)\frac{1}{\Delta\widehat{m}} =$$

$$\left(\frac{1}{\frac{1}{1-\frac{m_1}{m_2}+\frac{m_1}{m_2}\frac{\Delta X}{(P_{cp}-X_{1cp})}}-1}\right)\frac{1}{\Delta\widehat{m}} = \left(\frac{1}{\frac{1}{1-\frac{m_1}{m_2}\left(1-\frac{\Delta X}{(P_{cp}-X_{1cp})}\right)}-1}\right)\frac{1}{\Delta\widehat{m}}$$

$$\boldsymbol{K_{m\_t_p}} = \left(\frac{1}{\frac{1}{1-\frac{m_1}{m_2}\left(1-\frac{\Delta X_{i_{cp}}}{\Delta P_{cp}}\right)}-1}\right)\frac{1}{\Delta\widehat{m}} \qquad (17)$$

where ΔP_{cp} and $\Delta X_{i_{cp}}$ – the average engine thrust margin and the average increase of the induced drag for the speed range acceleration.

The value of the coefficient slightly depends on the percentage change in the mass. Assuming $\Delta X_{i_{cp}}/\Delta P_{cp} = 0$ (ΔX_i is significantly less than the average available thrust ΔP_{cp}) $K_{m\_t_p}$=1, and taking into account the influence of increased inductance (due to the increase in mass) the coefficient value will be slightly larger. For example, for Mig-29 during acceleration 600 – 1100 km/hour near the ground $K_{m\_t_p} = 1,12$ (Table 5).

Table 5: The values $K_{m\_t_p}$ for Mig-29 (600 – 1100 km/hour on the deck)

m_1, kg	Δm, %	ΔC_y	ΔCx_i	P_{cp}, kgf	X_{cp}, kgf	ΔX_i, kgf	$K_{m\_t_p}$
15000	10	0.011	0.001	16000	4000	130	**1.12**

If accept $\Delta Cx_{0cp} \approx \Delta Cx_0$ and $Cx_{01cp} \approx Cx_{0_1}$, $K_{Cx_0\_t_p}$ takes the following form:

$$K_{Cx_0\_t_p} = \frac{\frac{t_{p2}}{t_{p1}}-1}{\frac{Cx_{0_2}}{Cx_{0_1}}-1} = \left(\frac{t_{p2}}{t_{p1}}-1\right)\frac{1}{\Delta\widehat{Cx}_0} = \left(\frac{1}{-1-\frac{(n_{xa})_{cp1}}{\Delta(n_{xa})_{cp}}}\right)\frac{1}{\Delta\widehat{Cx}_0} =$$

$$\langle\Delta(n_{xa})_{cp} = \frac{-\Delta X_0}{G_1}\rangle = \left(\frac{1}{\frac{P_{cp}-X_{1cp}}{\Delta X_{0cp}}-1}\right)\frac{1}{\Delta\widehat{Cx}_0} =$$

$$\left(\frac{1}{\frac{P_{cp}-X_{1cp}-\Delta X_{0cp}}{\Delta X_{0cp}}}\right)\frac{1}{\Delta\widehat{Cx}_0} = \left(\frac{\Delta Cx_{0cp}qS}{qS\left(\frac{P_{cp}}{qS}-Cx_{01cp}-\Delta Cx_{0cp}\right)}\right)\frac{Cx_{0_1}}{\Delta Cx_0} =$$

$$\langle\begin{array}{l}\Delta Cx_{0cp}\approx\Delta Cx_0\\ Cx_{01cp}\approx Cx_{0_1}\end{array}\rangle = \frac{Cx_{0_1}}{\frac{P_{cp}}{qS}-Cx_{01cp}-\Delta Cx_{0cp}} = \frac{1}{\frac{P_{cp}}{X_{01cp}}-1-\Delta\widehat{Cx}_0}$$

$$K_{Cx_{0\_t_p}} = \frac{1}{\frac{P_{cp}}{X_{0_{1cp}}} - 1 - \Delta \widehat{Cx_0}} \qquad (18)$$

The coefficient value slightly depends on the percentage change Cx_0 and is associated with a specific aircraft. For Mig-29 $K_{Cx_{0\_t_p}} = 0.27$ (Table 6).

Table 6: The values $K_{Cx_{0\_t_p}}$ for Mig-29 (600 – 1100 km/hour near the ground)

Cx0	Δ Cx0, %	Pcp, kgf	$X_{0_{1cp}}$, kgf	$K_{Cx_{0\_t_p}}$
0.025	10	16000	3300	**0.27**

4. Calculation of the influent coefficients by numerical method

To confirm the derived dependencies (8), (10), (12), (14), (15), (17), (18), we calculate the IC using the "Program for preparing the initial data and calculating the FQP". This software product was developed by Sukhoi Design Bureau and verified by many years of the application practice. The program uses weight, aerodynamic characteristics ($Cx_0(M)$, Cx_i (C_y, M), $Cy_{max}(M)$, C_y^a)

and propulsive characteristics P (H, M, Dr) Qc(H, M, Dr), as well as some limitations (q_{max}, ny_{max}) and constants. The graphs $Cx_0(M)$, Cx_i (C_y, M), $Cy_{max}(M)$ C_y^a, P (H, M), Qc(H, M) are digitized from books of practical aerodynamics [18-21]. ICs are calculated according to the algorithm shown in the Figure 1.

We calculate IC for the following aircraft of the class of operational - tactical aviation: fighter 4th generation Su-27; frontline bomber Su-24; strike-fighter Su-25; fighter 4th generation Mig-29; fighter 3rd generation Mig-23; interceptor jet 3rd generation Mig-25.
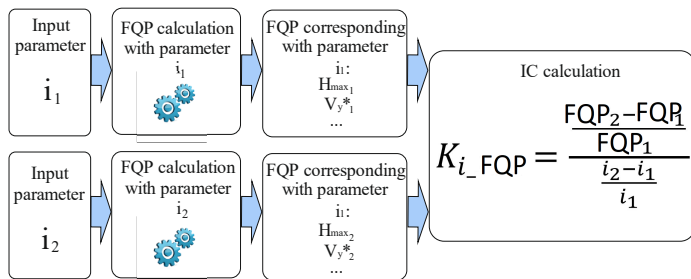
Figure 1: IC calculation scheme

Table 9 and Figures 2-11 show the results of calculating the IC programmatically. ICs were obtained when the mass changed by 10%, Cx_0 changed for M<1 by 10% and for M>1 by 18%. Such differences in the spread of Cx_0 are taken on the assumption that the inaccuracy of determining this parameter for the supersonic domain of flight is higher. Aircraft characteristics are taken in the calculations are presented in Table 7.

The effect on the service ceiling is shown in Figures 2 and 3, on the specific excess power - in Figures 4 and 5, on the sustained load factor - in Figures 6 and 7, for the acceleration time - in Figures 8 and 9. Figures 2-9 also show the values obtained from the above formulas.
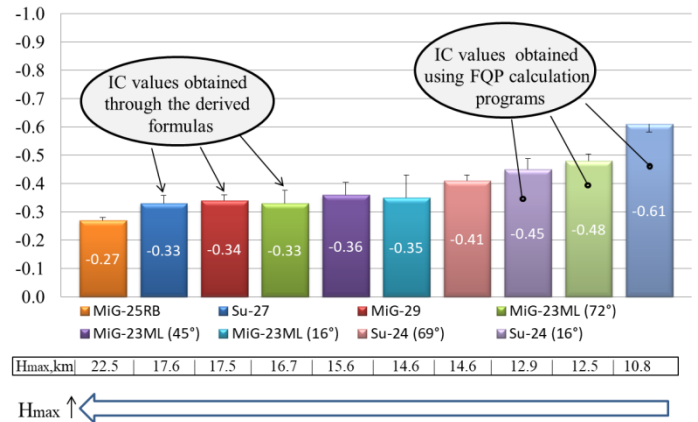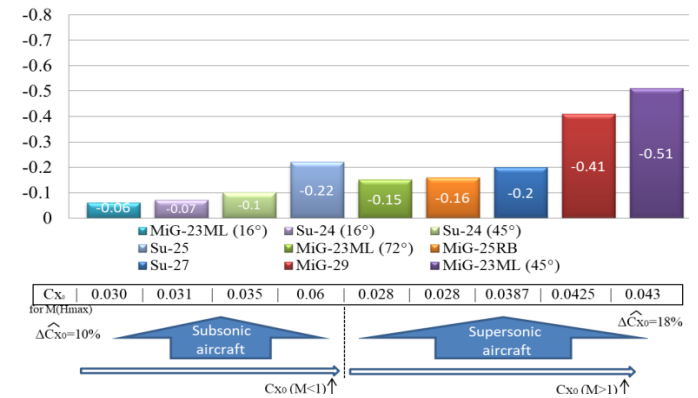

Figure 2: $K_{m\_H_{max}}$ values

Figure 3: $K_{Cx_{0\_H_{max}}}$ values

Table 7: Aircraft characteristics, used in the calculations

Aircraft	MiG-29	MiG-25RB	MiG-23ML (72°)	MiG-23ML (45°)	MiG-23ML (16°)	Su-27	Su-25	Su-24 (69°)	Su-24 (45°)	Su-24 (16°)
Weight										
Mass in calculations, kg	15000	24000	18000	18000	18000	21200	13000	25000	25000	25000
Wing parameters										
Wing area, m²	38	61.5	34.16	35.3	37.27	62	30.1	51	53	55.18
Wing extension	3.39	2.94	1.77	3.43	5.26	3.5	6	2.11	3.9	5.64
Leading edge a sweep angle, deg.	42	41	74	47	18	42	20	69	45	16
Power plant specifications										
Jet engine	2xRD-33	2xR-15-300	R-35	R-35	R-35	2xA1-31F	2xR-95 SH	2xA1-21 F-3	2xA1-21 F-3	2xA1-21 F-3
Thrust (H=0, M=0), kgf	16000	22400	12500	12500	12500	25000	8200	22400	22400	22400
Aircraft design parameters										
Starting thrust-to weight ratio	1.07	0.93	0.69	0.69	0.69	1.18	0.63	0.90	0.90	0.90
Wing load, kg/m²	395	390	527	510	483	342	432	490	472	453

Table 8: IC for the studied aircrafts

FQ&P	Su-27 m	Su-27 Cx0	MiG-29 m	MiG-29 Cx0	MiG-23ML (72°) m	MiG-23ML (72°) Cx0	MiG-23ML (45°) m	MiG-23ML (45°) Cx0	MiG-23ML (16°) m	MiG-23ML (16°) Cx0
Vmax	-0.07	-0.81	-0.09	-0.71	-	-	-0.14	-0.85	-0.01	-0.17
Hmax	-0.33	-0.2	-0.34	-0.41	-0.33	-0.15	-0.36	-0.51	-0.35	-0.06
Vy*	-0.91	-0.37	-0.91	-0.47	-1.02	-0.47	-0.98	-0.53	-0.91	-0.45
ny	-0.91	-0.09	-0.91	-0.09	-0.91	-0.11	-0.91	-0.1	-0.91	-0.09
tp	1.09	0.26	1.08	0.27	1.22	0.24	1.11	0.28	1.02	0.35

FQ&P	MiG-25RB m	MiG-25RB Cx0	Su-25 m	Su-25 Cx0	Su-24 (69°) m	Su-24 (69°) Cx0	Su-24 (45°) m	Su-24 (45°) Cx0	Su-24 (16°) m	Su-24 (16°) Cx0
Vmax	-	-	-0.02	-0.24	-	-	-0.02	-0.18	-0.02	-0.14
Hmax	-0.27	-0.16	-0.61	-0.22	-0.41	-	-0.48	-0.1	-0.45	-0.07
Vy*	-0.96	-0.43	-0.94	-0.46	-0.98	-0.39	-0.96	-0.45	-0.94	-0.41
ny	-0.91	-0.21	-0.91	-0.26	-0.91	-0.19	-0.91	-0.14	-0.91	-0.18
tp	1.06	0.35	1.05	0.35	1.19	0.31	1.11	0.22	1.06	0.30

- subsonic aircraft

Sustained load factor on high 1000 m M=0.8 (for Su-25 M=0.6)

Acceleration time on the deck 600-1100 km/hour (for Su-25 500-800 km/hour)
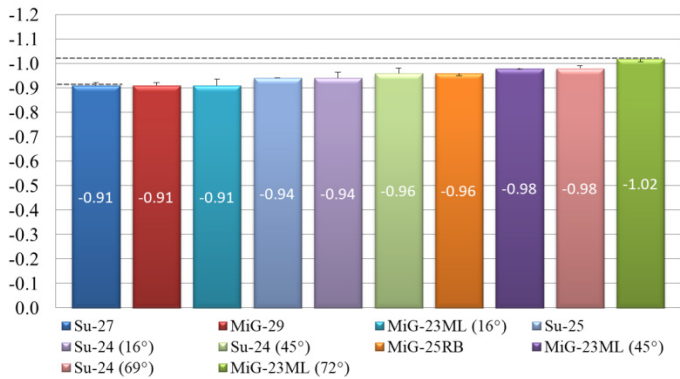
Aircrafts images courtesy of Google images
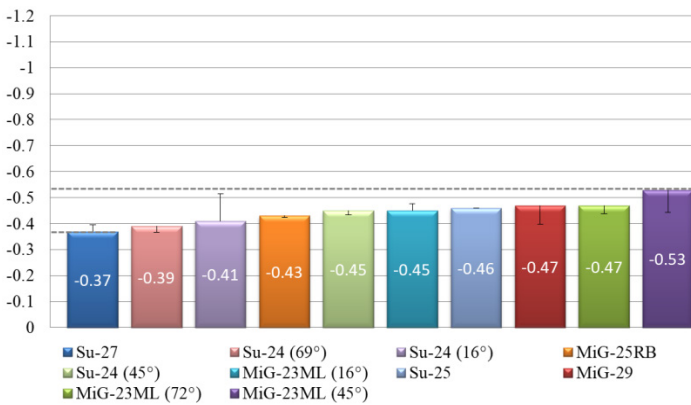

Figure 4: $K_{m\_V_y^*}$ values
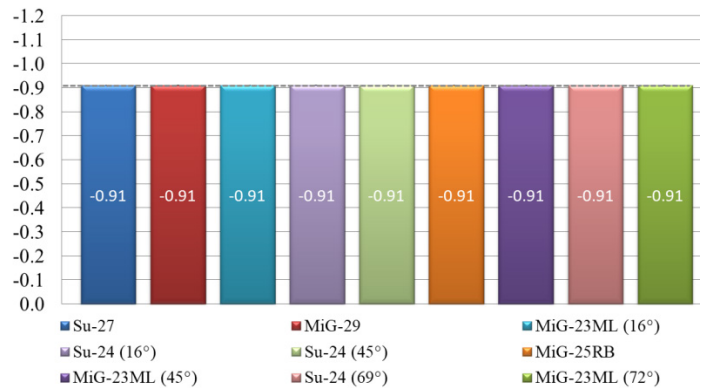

Figure 6: $K_{m\_n_y}$ values (H=1000m, M=0.8)

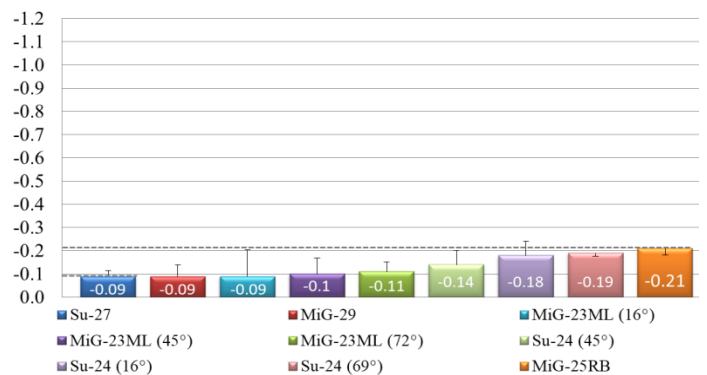Figure 5: $K_{C_{x_0}\_V_y^*}$ values

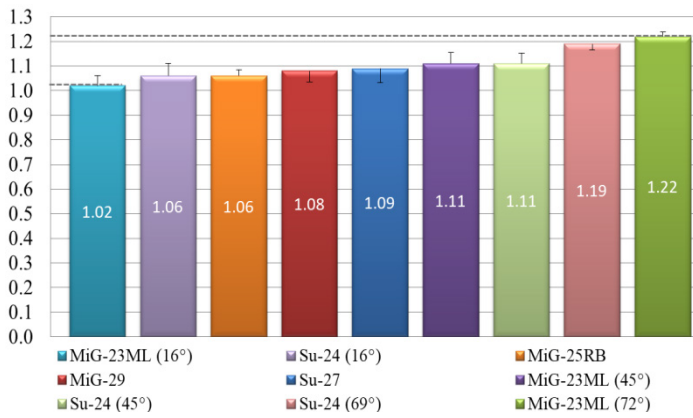Figure 7: $K_{C_{x_0}\_n_y}$ values (H=1000m, M=0.8)
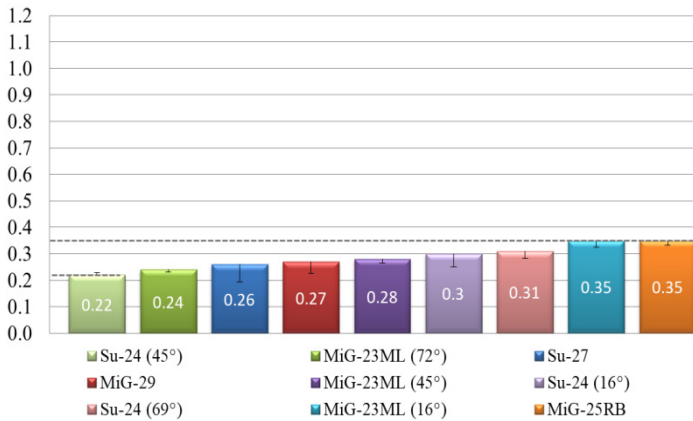
Figure 8: $K_{m\_t_p}$ values

Figure 9: $K_{C_{x_0}\_t_p}$ values

5. Results and Discussions

As can be seen from Figures 2-10, the analytically obtained ICs are confirmed by numerical calculations. The absolute error when using the derived formulas is less than 0.1 of the IC value, which in terms of FQP values gives an error of less than 10%. At the initial stage of aircraft design (when the error of other constituent elements is comparable in magnitude), this is an acceptable indicator.

Summarize the calculated IC in Table 9, the percentage change in FQP is presented in Table 10.

Table 9: IC ranges for the considered OTA aircrafts

FQP	K_{m_FQP}	$K_{Cx_0\_FQP}$	
		$M_{max}<1$	$M_{max}>1$
V_{max}	-0.1	-0.1…-0.25	-0.7…-0.85
H_{max}	0.25…0.6	-0.05…-0.2	-0.15…-0.5
$V^*_{y\,max}$	-0.9….-1.05	-0.35…-0.55	-
n_y (H=1000m, M=0.8)	-0.91	-0.05…-0.2	-
t_p (600-1100 km/h, H=200m)	1…1.25	-0.2…-0.35	-

Table 10: Percent change in the FQP

FQP	With an increase in mass by 10%, the FQP deteriorates by:	With an increase in Cx_0 in the subsonic area by 10% and by 18% in supersonic the FQP deteriorate by:	
		$M_{max}<1$	$M_{max}>1$
V_{max}	1%	1…2.5%	12…15.5%
H_{max}	decreases by 630 m	0.5…2%	3…8%
$V^*_{y\,max}$	9….10.5%	3.5…5.5%	-
n_y (H=1000m, M=0.8)	9.1%	0.5…2%	-
t_p (600-1100 km/h, H=200m)	10…12.5%	2…3.5%	-

From Tables 9 and 10 we can conclude that the considered FQP, such as the maximum specific excess power, sustained load factor, and acceleration time, are most sensitive to a change in mass. The inaccuracy in determining the mass in 1% gives a spread of these FQP in about 1% (more accurate values are given earlier). The IC of the mass on the practical ceiling depends on the absolute value of the ceiling, so it makes sense to talk about its absolute change. With an inaccuracy of mass of 1%, the ceiling is determined with an accuracy of about 60-70 meters.

To inaccuracy C_{x_0} is most sensitive to the maximum flying speed (for M>1). The numerical value of the IC on V_{max} is determined by the curve crossing nature of the required and available thrust (Figure 11). Graphs Cx_0 (M) for the studied aircrafts are shown in Figure 10.
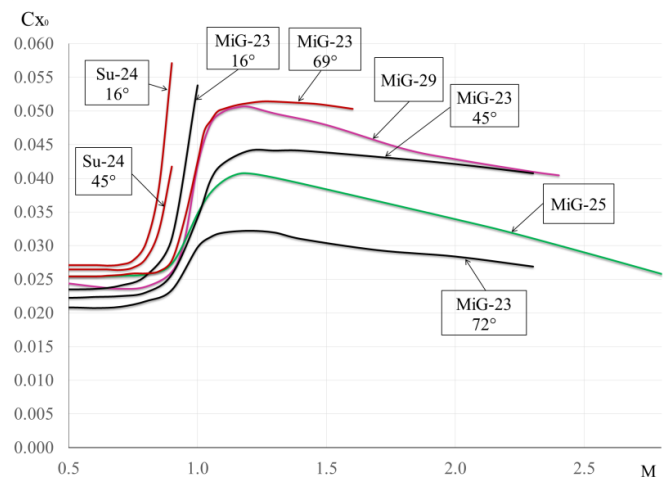

Figure 10: Graphs Cx0 for the studied aircrafts.

Figure 10 shows that in the area of maximum flight speed, the slope of the curves C_{x0} (M) is different for different aircraft.

The power-available curve depends on the engine unit's altitude speed performances, the required one on the drag run, therefore, the IC depends strongly on the specific aircraft (on the flow pattern of Cx_0 and altitude-speed performances), therefore,

linking this coefficient to the aircraft parameters is impractical. However, as the research showed, the IC value for subsonic aircraft is approximately 0.1…0.2 (Table 9), for supersonic: 0.7…0.9.
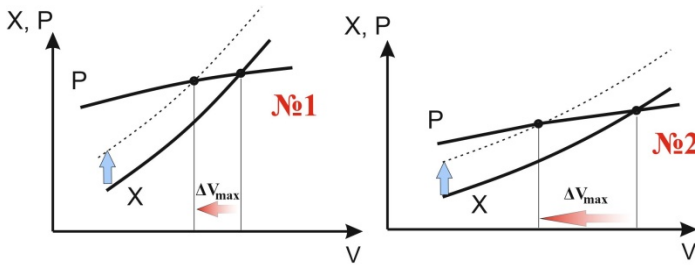

Figure 11. The different nature of Zhukovsky curve crossing. P – engine thrust curve, X – airplane drag curve.

6. Conclusions

The results of this work allow us to estimate the contribution of mass uncertainties and drag coefficient at zero lifting force to the range of variation of the FQP values.

The formulas obtained make it possible to determine the quantitative influence of the parameter on the value of the FQP. Verification of the formulas for some aircraft from the class of operational-tactical aviation showed that the error in determining the spread of flight characteristics according to the formulas obtained is within 10%.

The research showed that the mass influence coefficients Cx_0 on maximum specific excess power, sustained load factor and acceleration time practically do not depend on the parameters of the considered aircrafts of the same class. The IC of mass and Cx_0 on the maximum speed and practical ceiling associated with specific aircraft parameters. The sensitivity of the maximum speed to the zero lift-drag coefficient is determined by the curve crossing nature of the required and available thrust.

The inaccuracy in determining the mass in 1% gives a spread of maximum specific excess power, sustained load factor and acceleration time in about 0.9…1.3%. For the studied aircrafts the inaccuracy in determining the zero lift-drag coefficient of 1% gives a spread in the maximum speed within 0.7 ... 0.9% for supersonic aircraft and 0.1 ... 0.2% for subsonic aircraft.

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