Advances in Science, Technology and Engineering Systems Journal
Vol. 2, No. 3, 160-170 (2017)
www.astesj.com
Special Issue on Recent Advances in Engineering Systems

ASTES Journal
ISSN: 2415-6698

A synchronizing second order sliding mode control applied to decentralized time delayed multi–agent robotic systems: Stability Proof

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A R T I C L E I N F O

Article history:
Received: 04 March, 2017
Accepted: 07 April, 2017
On-line: 20 April, 2017

Keywords :
Sliding Mode Control (SMC)
Synchronization
multiple robot manipulators
Cross Coupling
time delay
stability

A B S T R A C T

This study investigates the synchronization issue of multiple robot manipulators in the presence of time delay. Since several previous works based on synchronization task neglect the communication delay, in this paper we develop a finite time stability based on a Lyaponov functional for synchronization of a networked robotic system where time delay exists during the communication between robots. To this effect, we consider a second order sliding mode control (SMC) combined with the cross coupling concept in order to ensure the position synchronization of networked robot manipulators. Furthermore, the stability of the proposed controller with communication's delay has been proved. Simulation results illustrate satisfactory performances which prove the efficiency of the proposed approach.

1 Introduction

Over the past few decades, an increasing interest has been noticed on the interconnected systems in several fields of research [1, 2]. In fact, cooperative and coordinated control have attracted several research communities such as: biology [3]; artificial intelligence [4]; wireless sensor networks[5, 6]; control of mobile robot[7, 8]; spacecraft [9, 10]. The synchronized control of robot manipulators has been firstly presented in [11, 12]. Subsequently, further research results on the synchronization of robot manipulators have been published [13, 14]. It is recognized that the existing synchronization works as aforementioned are all for motion control. In this context, motion synchronization of multi agent systems has attracted much attention in various applications including the industrial assembling, automatic control of multi agent systems such as the control of robot manipulators [15, 16].

As an illustration, in [17] position synchronization of multiple motion axes has been studied. Motion synchronization has also been used in more complex mechanical systems especially the nonlinear robot systems. Furthermore, where flexibility and maneuverability are highly recommended [13, 18, 19], with the increasing complexity of evolved and specific applications, manipulability can't be fulfilled by a simple robot. For this reason, the use of cooperative schemes for multiple robots can present a better solution to realize more robust multi agent system controls, where each robots operates cooperatively, and receives feedbacks from each others to achieve a consolidated goal [11, 20].

Furthermore, most of the real systems are known by nonlinearities such as robotic field. For this reason, the formulation for robust control laws is required,in the sense that it is able to ensure the system stability and the robustness via external disturbances and parameters variations [21, 22]. Otherwise, several control methods have been used to synchronize various complex systems such as: adaptative control [23, 24]; sliding mode control(SMC) [25, 26, 27]; neural networks [28] etc.

Among several dynamic behaviors, the synchronized motion control and the stability of complex nonlinear systems are considered between the most important research topics during several years [29, 30]. The mutual synchronization of robotic systems without time delay has been absolutely studied by several communities [11, 13]. Nevertheless, and due to the importance of cooperative research using multiple agents and within its wide range applications, the control algorithm suffers from some disturbing factors that can't be neglected such as time delays communication, communication interruption, packet collisions etc. Time delay communication is commonly

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https://dx.doi.org/10.25046/aj020321

known in biological and physical networks, owing to the finite velocity of communication as well as traffic congestions [31, 32, 33]. In multi–agent robotic systems, the time delay is only considered at the level of the information arriving to a robot and coming from sensors of its neighbors, while information from its own sensors is processed immediately.

The presence of this communication constraint makes the dynamical behaviours more complicated, can lead to undesirable transient response, reduces the performances of the networked systems or even the instability of the system [33, 34, 35]. Referring to the cross-coupling technique, several works have been suggested to improve the performances of the synchronization of multi axis motions [36, 37]. Later on, [38] proposed a control algorithm in order to synchronize networked systems in the presence of time delays [39, 40]. In the light of what was said, we present in this paper a synchronizing SMC algorithm for the control of a time delayed multi agent system in order to prove the effectiveness and the stability of the proposed approach.

1.1 Contribution

The subject of this paper is to realize the motion control of complex networked robotic systems in the presence of time delays.
Since the robot requires an interaction with its environment to achieve its goal, and in the absence of a suitable sensor, the robot remains blind. For this reason, and in order to reproduce human capacities for perception and action in robotic systems, researchers adopt the integration of data from a surveillance camera.
This camera is a great way to provide security to the target location. Nowadays, the surveillance cameras can also be set to be motion activated, recording footage when motion triggers them. Many range of cameras also includes outdoor security cameras and wifi cameras in order to effectively control the monitoring task.
Therefore, a second order sliding mode strategy has been considered and exhibited on a 3 degrees of freedom($3DOF$) surveillance camera system, where we focus on the manipulative arm managing the camera movements.
Then, to guarantee the overall vision of the proposed framework, we combine a muli robot manipulators, where each robot must synchronize its movement with other teammates using the cross-coupling approach .
The main goal of this work is to realize a common and performant motion control task of multi-agent robot manipulators based on the cross coupled second order sliding mode approach design, by reducing the chattering impact, and achieving robust communication between agents which make the system stronger against disturbances, uncertainties, breakdowns and also able to compensate the existence of communication delay.

2 Second Order Sliding mode controller

2.1 Preliminaries

Sliding mode control is a robust nonlinear strategy [41, 46, 47]. Such Variable Structure Control (VSC) is considered as a discontinuous feedback approach where its design is divided into two parts: the reaching phase (system trajectories are forced to reach a specific surface in the state space then to remain on it) and the sliding phase (Figure 1).
The Sliding Mode approach is developed using the Lyaponov Theory in order to ensure the convergence to the sliding surface ($s(x) = 0$):

$$S^T \dot{S} < 0 \tag{1}$$

where the sliding surface S is chosen as:

$$S(x) = Y(x - x_d) \tag{2}$$

in which: x_d is the desired trajectory and Y is a matrix chosen such away x acheives x_d.
The structure of the proposed controller is composed of two terms:

$$u \quad = \quad u_{eq} + \Delta u \tag{3}$$

where u_{eq} is the equivalent control which ensure the ”reaching phase” and Δu is the corrective term used to avoid all deviations from the sliding surface.
The expression of the equivalent term can be deduced from the following equation:

$$\dot{S} = F(x) + G(x)u = 0 \quad \mapsto \quad u_{eq} = -[G(x)]^{-1} F(x) \tag{4}$$

where $F(x)$ and $G(x)$ are defined by the affine state equation of a nonlinear system ($\dot{x} = f(x) + g(x)u$).
Moreover, the corrective term can be described as follows:

$$\Delta u \quad = \quad -[G(x)]^{-1} W \text{sign}(S) \tag{5}$$

where W is a definite positive matrix.

2.2 Second Order Sliding Mode Control

As it is mentioned above, this robust approach suffers from the undesirable chattering phenomenon [14, 34] induced by the corrective term Δu, whose impact is manifested by the existence of perturbing high switching frequencies in the control inputs [21, 42]. More precisely, this problem involves fast and sudden changing control signals which lead to low accuracy and even damage the mechanical parts. Therefore, several methods have been developed in order to overcome this annoying phenomenon [22].
In this context, the second order sliding mode approach presents an enhancement of the classical SMC by the introduction of a filtering action in the controller.
Such an action greatly reduces the major drawback of

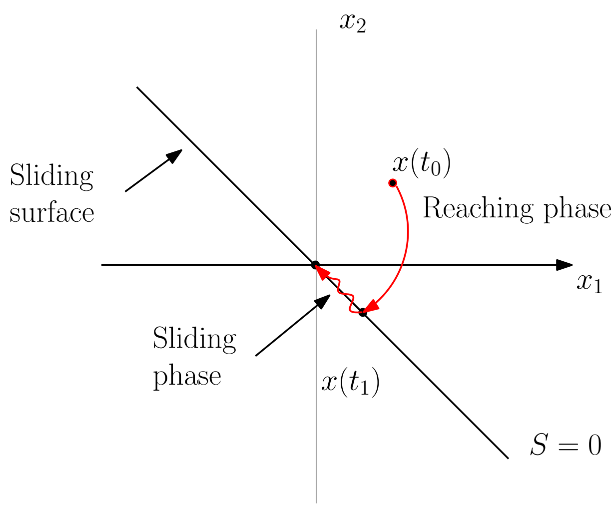


Figure 1: Phases of Sliding Mode Control

the simple SMC which is the chattering phenomenon. Hence, the second order SMC has been considered as the most useful among the high order SMC thanks to its relative simplicity of application, compared to the higher order controls [43, 44, 45].
Then, the sliding surface derivative of the second order SMC approach has been modified as follows :

$$\dot{s_i} = \sigma_i \quad \mapsto \quad \Delta u_i = [Wg(x)]^{-1}\sigma_i \tag{6}$$

The new description of the dynamic control behavior can be written as follows:

$$\begin{cases} \dot{s_i} &= \sigma_i \\ \dot{\sigma_i} &= -a_0\, s_i - a_1\, \sigma_i + v_i \end{cases} \tag{7}$$

where a_0, a_1 are positive scalars and v_i is a variable control of SMC. This study presents multi-input-multi-output systems. Furthermore, the expression of $\dot{\sigma_i}$ can be deduced from the equality $A(p) = 0$ of the following Hurwitz polynomial (which its roots have negative real parts) :

$$A(p) = (p + \mu)^2 \tag{8}$$

where μ is a positive scalar.
In order to ensure the stability, the representation (7) can be reformulated as the condensed form:

$$\dot{Z_i} = \phi Z_i + \Gamma v_i \tag{9}$$

in which:

$$\underbrace{\begin{pmatrix} \dot{s_i} \\ \dot{\sigma_i} \end{pmatrix}}_{\dot{Z_i}} = \underbrace{\begin{pmatrix} 0 & I \\ -\mu^2 I & -2\mu I \end{pmatrix}}_{\phi} \begin{pmatrix} S_i \\ \sigma_i \end{pmatrix} + \underbrace{\begin{pmatrix} 0 \\ 1 \end{pmatrix}}_{\Gamma} v_i \tag{10}$$

where 0 is the null matrix, I is the identity matrix and the discontinuous term v_i is given by .

$$v_i = -Q\ \text{sign}\ (\Gamma^T L S_i) \tag{11}$$

where $Q = [q_1, q_2, ..., q_n]$ and L are positive definite matrix.

3 Mutual SMC synchronization algorithm

3.1 Mathematical model

A robotic manipulator arm designed to be equipped with a surveillance camera system presents the adopted dynamic model of this study (Figure 2).

Using the Lagrangian formulation, the motion equation of a manipulator robot "i" can be written as: [46]

$$M_i(q_i)\ddot{q_i} + C_i(q_i, \dot{q_i})\dot{q_i} + G_i(q_i) = \tau_i \tag{12}$$

where:

- $q_i(t) \in \mathbb{R}^n$ is the measured articulation vector of the manipulator (joint position),
- $\dot{q_i} \in \mathbb{R}^n$ is the velocity vector,
- $\ddot{q_i} \in \mathbb{R}^n$ is the joint acceleration vector,
- $M_i(q_i) \in \mathbb{R}^{n\times n}$ is the symmetric uniformly bounded and positive definite inertia matrix,
- $C_i(q_i, \dot{q_i})\dot{q_i} \in \mathbb{R}^n$ represents the vector expressing Coriolis and centrifugal forces,
- $G_i(q_i) \in \mathbb{R}^n$ is the vector of gravitational torques,
- $u = \tau_i \in \mathbb{R}^n$ denotes the control torque.

3.2 Cross Coupling technique

In this work, we take into consideration the synchronization of multiple robot manipulators. In this context, we propose decentralized control laws for n robots manipulators for which each robot synchronizes its position with the other neighbor agents and track the same desired trajectory. Specifically, using the synchronization approach, manipulators are controlled in a synchronous manner so that the tracking errors and the synchronization errors converge

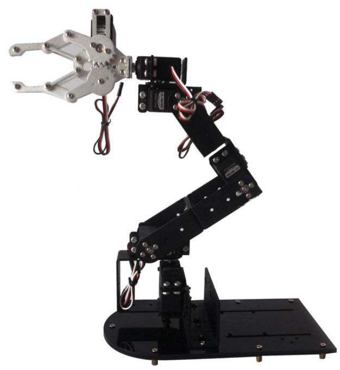


Figure 2: Example of $3DOF$ robot manipulator

to zero where the synchronization error is considered to be a differential position between coordinated agents. In order to achieve a coordinated control motion, an effective synchronization method namely the Cross Coupling concept is carried out where the whole multi-robot manipulators is used as a unique generalized system [47, 48, 49].
The cross coupling design was firstly introduced by [2], then its concept has been used mainly for machine tools [50, 51]. Later on, it has been applied in the robotic fields by [52]. The main idea of such a procedure is to create a global error of the model [36, 53].
In the light of what was said, the tracking error can be written as follows:

$$\eta_i(t) = q_i(t) - q_d(t) \tag{13}$$

where $q_d(t) \in \mathbb{R}^n$ denotes the desired position.
The vector η_i will give insight on the joint positions convergence to the desired trajectory. The proposed cross coupling concept suggests a suitable synchronization error which is defined as follows:

$$\begin{aligned} \xi_i(t) &= \textstyle\sum_{j\neq i}^{p} \Lambda_{ij}[q_i(t) - q_j(t-\tau)] \\ &= \sum \Lambda_{ij}(q_i(t) - q_d(t)) - \sum \Lambda_{ij}(q_j(t-\tau) - q_d(t-\tau)) \\ &\quad + \sum \Lambda_{ij}(q_d(t) - q_d(t-\tau)) \\ &= \sum \Lambda_{ij}\eta_i - \sum \Lambda_{ij}\eta_j(t-\tau) + \sum \Lambda_{ij}[q_d(t) - q_d(t-\tau)] \end{aligned} \tag{14}$$

where Λ_{ij} is a symetric positive definite matrix which reveals an idea about the communication quality between the i^{th} and j^{th} agents.
Thus, in order to attain a robust controller for multi robot systems, and to ensure a synchronous trajectory tracking in the presence of communication data delay, we define the following global error expression (for robot i):

$$\varepsilon_i = \eta_i + \int_{t_0}^{t} \xi_i(\alpha)d\alpha \tag{15}$$

where we note that this error expression includes both synchronization error and trajectory tracking error defined above.
Then, its derivative can be considered as follows:

$$\dot{\varepsilon}_i = \dot{\eta}_i + \xi_i \tag{16}$$

Let's define the sliding mode surface :

$$s_i = \dot{\varepsilon}_i + \lambda_i \varepsilon_i \tag{17}$$

where $\lambda_i > 0$. Then, we obtain:

$$\dot{\eta}_i = -\xi_i - \lambda_i \varepsilon_i + s_i \tag{18}$$

Having in mind the expression of (14), we obtain the following expression:

$$\begin{aligned} \dot{\eta}_i &= -\sum \Lambda_{ij}\eta_i + \sum \Lambda_{ij}\eta_i(t-\tau) \\ &\quad -\sum \Lambda_{ij}[q_d(t) - q_d(t-\tau)] - \lambda_i\varepsilon_i + s_i \end{aligned} \tag{19}$$

In order to simplify the previous expression, we define:
$d_i = \sum \Lambda_{ij}[q_d(t) - q_d(t-\tau)]$.
Thus:

$$\dot{\eta} = A\eta + B\eta(t-\tau) + d - \Lambda\varepsilon + S \tag{20}$$

where:

$$A = \begin{pmatrix} -\sum \Lambda_{1j} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & -\sum \Lambda_{nj} \end{pmatrix}, \quad \eta = \begin{bmatrix} \eta_1 \\ \eta_2 \\ \vdots \\ \eta_n \end{bmatrix}$$

$$B = \begin{pmatrix} 0 & \sum \Lambda_{12} \cdots & \sum \Lambda_{1n} \\ \sum \Lambda_{21} & \sum \Lambda_{23} \cdots & \sum \Lambda_{2n} \\ \vdots & \ddots & \vdots \\ \sum \Lambda_{n1} & \cdots & 0 \end{pmatrix}, S = \begin{pmatrix} s_1 \\ s_2 \\ \vdots \\ s_n \end{pmatrix}$$

$$d = \begin{pmatrix} d_1 \\ d_2 \\ \vdots \\ d_n \end{pmatrix}, \Lambda = \begin{pmatrix} \lambda_1 I & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \lambda_n I \end{pmatrix}$$

The equivalent control expression can be deduced from the equality $\dot{s}_i = 0$ then, we obtain:

$$u_{eq_i} = M_i[\ddot{q_d} - \dot{\xi}_i + \lambda_i(\xi_i + \dot{\eta}_i)] + C_i(q_i, \dot{q}_i)\dot{q}_i + g_i(q_i) \tag{21}$$

3.3 Theorem

The control law:

$$u = u_{eq_i} + \Delta u \tag{22}$$

where: u_{eq_i} is expressed in equation (21) and Δu is given by:

$$\Delta u = NR^{-1}[N\varepsilon - P\eta - \Lambda S - K sign S] \tag{23}$$

with N is a diagonal, symmetric definite positive matrix which stabilities the proposed system composed by n manipulators.
Remembering that for the Second Order Sliding Mode Control, the sliding surface becomes $\dot{s}_i = \sigma_i$ and taken into consideration equation (17), we obtain:

$$\ddot{q}_i - \ddot{q}_d + \sum \Lambda_{ij}(\dot{q}_i - \dot{q}_j) + \lambda_i[\dot{q}_i - \dot{q}_d + \sum \Lambda_{ij}(q_i - q_j)] = \sigma_i \tag{24}$$

Retaking the mathematical model equation (12) and substituting it in the previous equation gives:

$$\begin{aligned}\sigma_i = M_i^{-1}[\tau_i - C_i(q_i, \dot{q}_i)\dot{q}_i - G_i(q_i)] \\ + \sum \Lambda_{ij}(\dot{q}_i - \dot{q}_j) + \lambda_i[\dot{q}_i - \dot{q}_d + \sum \Lambda_{ij}(q_i - q_j)] - \ddot{q}_d\end{aligned}$$

And consequently,:

$$\begin{aligned}\tau_i = C_i(q_i, \dot{q}_i)\dot{q}_i + G_i(q_i) + M_i[\ddot{q}_d - \sum \Lambda_{ij}(\dot{q}_i - \dot{q}_j) \\ -\lambda_i[(\dot{q}_i - \dot{q}_d) + \sum \Lambda_{ij}(q_i - q_j)] + \sigma_i]\end{aligned} \tag{25}$$

4 Proof of the Stability Analysis

In order to prove the stability of the considered multi agent system, a first Lyaponov function is chosen as follows:

$$V_1 = \eta^T P \eta \tag{26}$$

Its derivative yields:

$$\dot{V}_1 = \eta^T(PA + A^T P)\eta + 2\eta^T P(B\eta(t-\tau) + d - \Lambda\varepsilon + S) \tag{27}$$

The second function is expressed as:

$$V_2 = \int_{t-\tau}^{t} \eta^T H \eta d\alpha \tag{28}$$

where:

$$\dot{V}_2 = \eta^T H \eta - \eta(t-\tau)^T H \eta(t-\tau) \tag{29}$$

Then and in order to simplify the expression of $\dot{V}_1 + \dot{V}_2$, we firstly compute the sub –equation:
$2\eta^T PB\eta(t-\tau) - \eta(t-\tau)^T H\eta(t-\tau)$.

So, we denote that $\zeta_1 = \rho_1\eta(t-\tau) - \frac{1}{\rho_1}D^{-1}B^T P\eta(t)$

Consequently:

$$\begin{aligned}\dot{V}_1 + \dot{V}_2 = \eta^T(PA + A^T P)\eta + 2\eta^T P(d - \Lambda\varepsilon + S) \\ +\eta^T H\eta - \zeta_1 D\zeta_1 + \rho_1^2\eta^T PBD^{-1}B^T P\eta \\ +\frac{1}{\rho_1^2}\eta(t-\tau)^T D\eta(t-\tau) - \eta(t-\tau)^T H\eta(t-\tau)\end{aligned} \tag{30}$$

After that, we regroup all terms of the previous equation:

$$\begin{aligned}\dot{V}_1 + \dot{V}_2 = \eta^T(PA + A^T P + H + \rho_1^2 PBD^{-1}B^T P)\eta \\ -\zeta_1^T D\zeta_1 - \eta(t-\tau)^T(H - \frac{1}{\rho_1^2}D)\end{aligned}$$

where H and D are positive definite matrix while P, N are symetric positive matrix.
It's obvious from the previous equation that there are three terms relative to d, ε and S which should been developed. Consequently:

$$2\eta^T Pd = -\zeta_2^T\zeta_2 + \rho_2^2\eta^T\eta + \frac{1}{\rho_2^2}d^T P^2 d$$

in which:

$$\begin{cases}\zeta_2 = \rho_2\eta - \frac{1}{\rho_2}Pd \\ -2\eta^T\Lambda\varepsilon = -\zeta_3^T\zeta_3 + \rho_3^2\eta^T\eta\frac{1}{\rho_3^2}\varepsilon^T\Lambda^T\Lambda\varepsilon\end{cases}$$

where $\zeta_3 = \rho_3\eta - \frac{1}{\rho_3}\Lambda\varepsilon$.

In the sequel, we introduce new terms V_3 and V_4 to complete the stability verification of the proposed synchronized control schemes such that:

$$\begin{cases}V_3 = \varepsilon^T N\varepsilon \\ V_4 = S^T RS\end{cases} \tag{31}$$

The differentiation with respect of time gives:

$$\begin{cases}\dot{V}_3 = 2\varepsilon^T N(s - \Lambda\varepsilon) = -\varepsilon^T(N\Lambda + \Lambda^T N)\varepsilon \\ -2\varepsilon^T Ns \\ \dot{V}_4 = 2\dot{S}^T RS\end{cases} \tag{32}$$

Therefore, we develop the following terms:

$$\begin{cases}2\eta^T PS = -\zeta_4^T\zeta_4 + \rho_4^2\eta^T\eta + \frac{1}{\rho_4^2}S^T P^2 S \\ -2\varepsilon^T NS = -\zeta_5^T\zeta_5 + \rho_5^2\varepsilon^T\varepsilon + \frac{1}{\rho_5^2}S^T N^2 S\end{cases} \tag{33}$$

in which $\zeta_4 = \rho_4\eta - \frac{1}{\rho_4}PS$ and $\zeta_5 = \rho_5\varepsilon + \frac{1}{\rho_5}NS$.

Finally, the derivative of the global Lyaponov function
$V = V_1 + V_2 + V_3 + V_4$ yields:

$$\begin{aligned}\dot{V} = \eta^T(PA + A^T P + H + \rho_1^2 PBD^{-1}B^T P + \rho_2^2 I + \rho_3^2 I \\ +\rho_4^2 I)\eta - \zeta_1^T D\zeta_1 - \zeta_2^T\zeta_2 - \zeta_3^T\zeta_3 - \zeta_4^T\zeta_4 - \zeta_5^T\zeta_5 \\ -\eta(t-\tau)^T(H - \frac{1}{\rho_1^2}D)\eta(t-\tau) - \varepsilon^T(N\Lambda + \Lambda^T N \\ +\rho_5^2 I - \frac{1}{\rho_3^2}\Lambda^T\Lambda)\varepsilon + 2S^T(R\dot{S} - N\varepsilon + P\eta) + \frac{1}{\rho_2^2}d^T P^2 d\end{aligned}$$

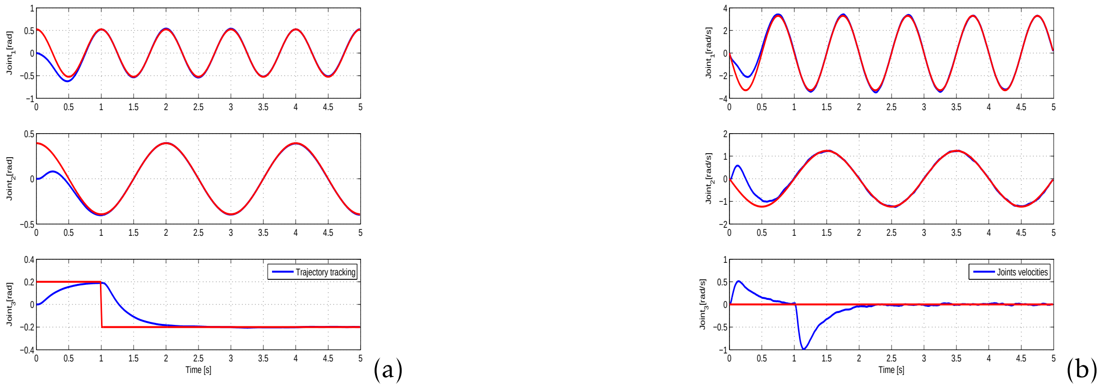


Figure 3: Simulation in the presence of external disturbances: (a) Positions evolutions in the presence of low measurement noises, (b) Velovities evolutions in the presence of low measurement noises

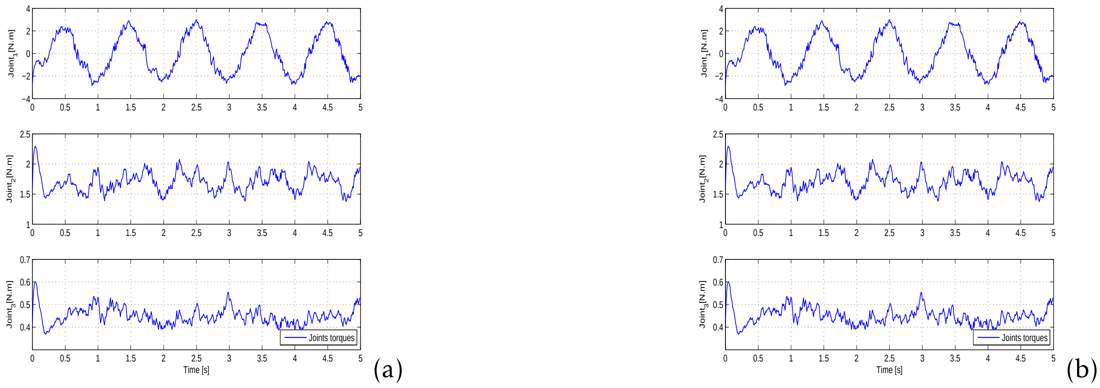


Figure 4: Simulation in the presence of external disturbances: (a) Torques evolutions of the SOSMC, (b) Measured torques evolutions of the SOSMC

Assume that $\dot{S}$ is defined as follows:

$$\dot{S} = R^{-1}[-N\varepsilon - P\eta - \Lambda S - K sign S] \tag{34}$$

Hence, remember that N, P, Λ, K are positive definite matrices, and assume that they verify the following conditions:

$$\begin{cases} H > \frac{1}{\rho^2} D \gg I \\ PA + A^T P + H + \rho_1^2 PBD^{-1}B^T P + (\rho_2^2 + \rho_3^2 + \rho_4^2) I \\ N\Lambda + \Lambda^T N > \frac{1}{\rho_3^2} \Lambda^T \Lambda \\ 2S^T(R\dot{S} - N\varepsilon + P\eta) + \frac{1}{\rho_2^2} d^T P^2 d = -2S^T \Lambda S < 0 \\ -2K \mid S \mid + \frac{1}{\rho_2^2} d^T P^2 d < 0 \end{cases}$$

The stability is confirmed if:
$\rho_1^2 PBD^{-1}B^T P \ll PA + A^T P$ and the term $\frac{1}{\rho_2^2} d^T P^2 d$ is considered as small bounded so that it can be neglected.
Consequently, we obtain:

$$\dot{V}_i \quad \leq 0$$

Finally, this confirms the stability of the overall system.

5 robustness via measurement noises effect

In order to test the robustness of the proposed controller via uncertainties, an additive measurement errors have been introduced.
Then the measured state can be expressed as follows:

$$x_m(t) = x(t)(1 + b_1(t)) = x(t) + \Delta x(t)$$

where b_1 is an additive bounded measure noise such that:
$\|b_1(t)\| \leq d_1$, and d_1 is a positive constant.
The level of the error effect has been varied from the lower impact to be gradually more intense, aiming to verify the controller's capacity to withstand such disturbances. At the beginning, low noise has been yield (we fluctuate the perturbation from 5 percent to 20 percent), we notice that the positions and the velocities evolutions still remain on the sliding surface which prove the robustness and the insensibility of the second order sliding mode control via disturbances (Figure 3). Then, the level has been increased (almost 30 percent), and in this case, the noise start to affect the tracking evolution of the system(Figure 5b and Figure 6b).
It is obvious from Figure 4 that there is a similarity between the real torques and the measured ones, this

Table 1: Joints parameters

Articulation	Mass	Length	Initial position
q_1	$2.7132(kg)$	$0.2(m)$	$\pi/6(rad)$
q_2	$1.1446(kg)$	$0.15(m)$	$\pi/4(rad)$
q_3	$0.3392(kg)$	$0.1(m)$	$0.2(rad)$

Table 2: Control parameters

Control Parameters	Values
Λ_{ij}	0.8
ω	20
$timedelay$	0.2

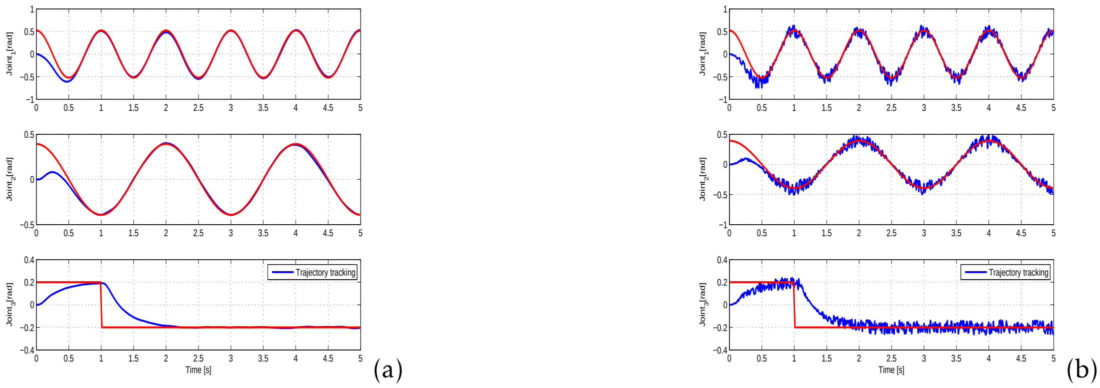


Figure 5: Simulations in the presence of external disturbances: (a) Positions evolutions in the presence of high measurement noises, (b) Measured Position evolutions in the presence of high measurement noises

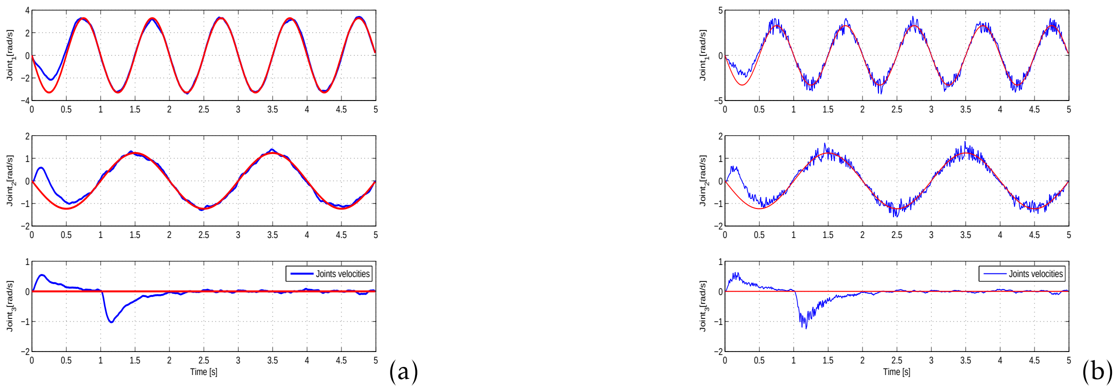
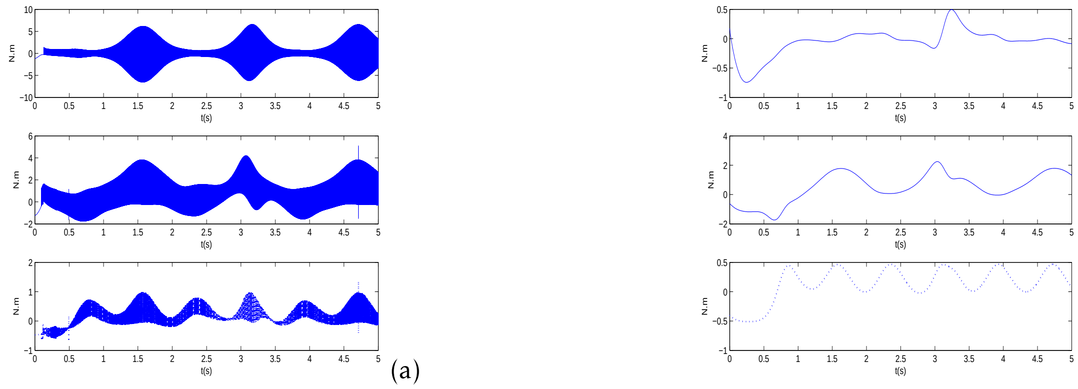


Figure 6: Simulations in the presence of external disturbances: (a) Velocities evolutions in the presence of high measurement noises, (b) Measured Velocities evolutions in the presence of high measurement noises

means that although the system suffers from external disturbances, the evolution of the applied torques do not record any notable increase, subsequently, no additional power consumption needed.

The aim of the comparison between real and measured simulations (Figure 5 and Figure 6) is to prove that the proposed controller exerts a satisfactory compensation action affecting the trajectory tracking and the velocity. This compensation becomes lower while increasing the disturbance effect, and it becomes unable to manage high imposed disruptions, i.e. starting from the presence of 30 present of errors applied to the system. Nevertheless, at this level the proposed controller has still resist to perturbations, and seen that the level of this disruption is relatively high, it can be then considered as a sufficient control, and proves its robustness via external disturbances.



(a) (b)

Figure 7: (a) The classical SMC torques evolutions, (b) The second order SMC torques evolutions

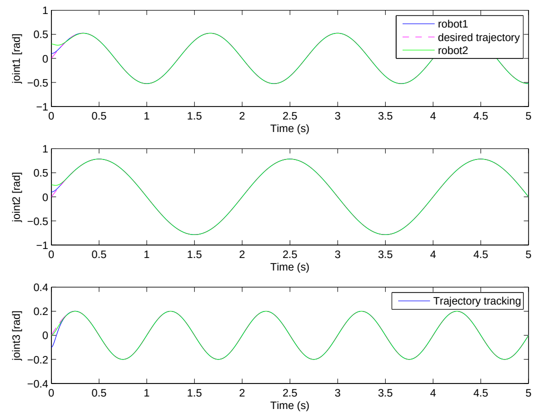


Figure 8: Motion Control and position synchronization

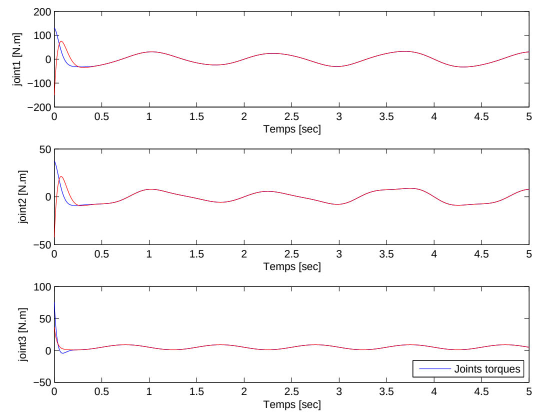


Figure 9: Evolution of the applied torques

6 Simulation Results

In this work, a second order SMC is applied to a networked multi–agent robotic system for a trajectory tracking control task. Indeed, the use of several cameras managed by manipulator robots and controlled by the proposed decentralized control law, allows the interaction of each robot with other agents in the networked system in order to make an overall vision about its environment. Thanks to the cross coupling concept, each agent is able to communicate and exchange information with its neighbors. The desired trajectory is expressed by:

$$q_d(t) = \begin{bmatrix} q_{d1}(t) \\ q_{d2}(t) \\ q_{d3}(t) \end{bmatrix} = \begin{bmatrix} \frac{\pi}{6}\sin(1.5\pi t) \\ \frac{\pi}{4}\sin \pi t \\ \frac{\pi}{2}\sin 4\pi t \end{bmatrix}$$

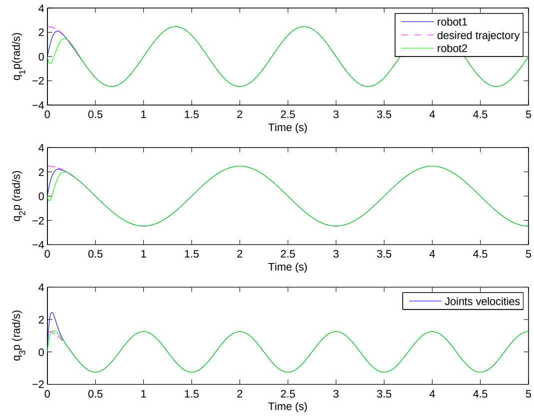


Figure 10: Velocity evolution of the synchronized robots

Parameters of the proposed system model used in simulation are illsutrated in Table 1 and control parameters values are given in Table 2.

Concerning the parameters μ_i and λ_i, they are defined as follows:

$$\lambda_i = 2\begin{pmatrix} 1.125 & & 0 \\ & 1.45 & \\ 0 & & 1.12 \end{pmatrix}$$

$$\mu_i = \begin{pmatrix} 3 & & 0 \\ & 4 & \\ 0 & & 5 \end{pmatrix}$$

Simulation results show the robust synchronization and the smooth evolution in the trajectory tracking based on the second order SMC. The original SMC torques evolutions and the second order SMC ones are shown in Figure 7*a* and Figure 7*b* respectively.

It is obvious from the Figure 7*a* that the classical sliding mode control suffers from the chattering phenomenon whose impact is reflected by the appearance of disrupting high switching frequencies (oscillations). Specifically, the problem consists of rapid and sudden changing control signals which lead to a low control accuracy. On the other side, Figure 7*b* demonstrates that the second order SMC seems to be smooth and able to reduce the chattering phenomenon.

Besides, the presence of time delay between the cooperative robots is slightly reflected during the simulation as shown in Figure 8 and Figure 10 respectively. It may be said otherwise that the communication time delay between robots is clearly compensated and the position synchronization based on the cross coupling concept is obvious, predominately in the (Figure 8). The presence of undesirable phenomenon namely chattering in control torques of each agent is avoided in Figure 9.

7 Conclusion

In this paper, the stability analysis of multi robot manipulator systems with a constant communication delay has been demonstrated. The main goal of this study is to compensate the delayed communication impact and to realize the synchronization between different robots of the system. So the proposed controlled system has succeeded to achieve a performing motion control task in the presence of loss of information during robot's communication, perturbations and also in the presence of delayed communication data. Simulation results show that the multi robots system can achieve the desired motion control task even with presence of a constant time delay.

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