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Selection of Rotor Slot Number in 3-phase and 5-phase Squirrel Cage Induction Motor; Analytic Calculation

Gábor Kovács *

Ganz Electric Works, 1095 Budapest, Hungary

A R T I C L E I N F O

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A B S T R A C T

With the spread of inverters, the attention of designers naturally turned to 5-phase motors, due to their advantageous properties. In this regard, perhaps the most important issue in the design of such machines is the selection of the correct rotor slot number. Many articles have been published on multiphase machines, however, only few of them deal with the rotor slot number. Although very useful results are achieved, the result is not comprehensive, systematic approach still awaits. In this paper, the behavior of the full range of 3-phase machines is calculated by using our formulas developed before, and then the method is simply transferred to 5-phase. The point is that relative rotor slot numbers are applied therefore the results are valid for any pole number on a comprehensive way. Detailed, well established design rules are provided. For better understanding of the process, the behavior of the oscillating torques generated by the synchronous parasitic torques during run-up and operation will be presented, using the equations of the unified electric machine theory. During the noise calculation, the concept of the Noise Component Equivalence Measure is introduced, which enables an accurate comparative calculation of the spectra of the radial force waves that excite the noise; it provides the final, comprehensive criteria for the selection of the right rotor slot number. With the study, the entire range of the slot number of any rotor mounted in any stator with any number of poles and number of slots is covered.

1. Introduction

The electrical machine designers noticed from the very beginning, that the MMF space harmonics occurring in the machine, and within that the harmonics created by the number of rotor slots, are of decisive importance for the machine's torque-speed characteristic curve, that means the resulting synchronous parasitic torques and the radial magnetic forces, the latter being the fundamental cause of noise generation.

Synchronous parasitic torques and radial magnetic forces, however, have always been examined separately, as the forced, namely "proportional" physical relationship between the two phenomena has not been recognized. The fact is, which slot number is prohibited or recommended from one point of view, it will also be from the other point of view and vice versa.

Numerous studies are cited at the end of the article as examples of today's research directions. Several articles deal with the issue of noise [1]-[4], others research the effect on the torque characteristic [5], [6]. A wide range of works investigate the multiphase machines, only a few works are highlighted here [3], [7]-[9]. Intensive research is also underway in order to clarify the fundamental laws of the torque ripple [10]-[12].

All these works achieve serious results with the help of significant apparatus. However, as far as the correct rotor slot number is concerned, each result only applies to a certain unique slot number *combination*, generality is lacking. Based on the investigations and tests carried out, so-called *slot number rules* were formulated, namely always for a ready stator. The basis of the phenomenon according to which every induction machine behaves in a way determined by the number of the rotor slots, the role of the number of the stator slot is only that it amplifies certain elements through its slot harmonics, has not appeared anywhere so far.

Our approach, however, so far and this time on is just the opposite, we *start from the rotor slot number* and then see what happens if that rotor will be installed in different stators. This is the way to achieve the general, comprehensive results below.

This paper is a synthetic, summary work, it strongly relies on our previous works, as a completion, in order to achieve the final

*Corresponding Author: Gábor Kovács, Ganz Electric Works, Hungary, gabor.kovacs.ganz@gmail.com

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goal, the right selection of the rotor slot number, therefore each of them should be put on the reference list ([13]-[18]).

Formulas not existed before were derived in [13], [15] and [18]. A general investigation on the influence of the relative rotor slot number on the behavior of the machine was carried out by using the new formulas in [14]. Detailed investigation was provided regarding parasitic torques and radial magnetic forces in [16] and [17]. The mutual correspondence of our formulas for the synchronous parasitic torques and the radial magnetic forces was proved also in [16], proving for the first time the direct link between the two phenomena. Experimental validation of the formula regarding synchronous parasitic torques was provided with the tests made by others in [17]. The effect of slot skew was investigated and new formulas were provided in [18].

The entire research is based on [19] as the very basic work of the electrical machine science history.

The tables for the three-phase machine developed before are now filled in by using our formulas with values of a “typical motor”. The introduction of the “typical motor” also contributed to the comprehensive approach, since in practice the relative breakdown torque and the relative starting current only move within a narrow band, consequently the reactance in p.u. are almost the same for any machine, for the entire range.

The procedure is then simply applied to the harmonics generated by the stator of 5-phase machines.

The 5-phase machines are powered almost exclusively via an inverter, where the synchronous parasitic torque occurring either in the motoric, in braking range or in standstill is of no importance; the ranges they appear in do not need to be distinguished. However, it is important that all of these appear as oscillating torques in operation, further no matter appearing in motoric or break range or in standstill, they cause radial magnetic forces of r=0 order.

It is simpler to formulate the aspects and performing the calculation for synchronous parasitic torques, therefore that investigation will be performed first. Then, in the second part of the study, it will be turned to the radial magnetic forces.

In the absence of our formulas developed before, the investigations carried out so far were forced to concentrate only on the expectedly dangerous frequencies and on how to avoid the coincidence, or to get involved in the use of large apparatus. However, now, relying on our formula, we have the opportunity to define and introduce what we call the Noise Component Equivalence Measure (NCEM). This metric made it possible to accurately calculate the comparative spectra of the complete, noise-exciting radial forces. Until now, such a method, such a table, which is more suitable than ever, yet very simple, which actually enables the design engineer to select the most favorable rotor slot number, indeed, has not been available.

As a model, such a machine was assumed for which the basic formulas for determining the resulting harmonics are valid [20]: infinite relative permeability, two-dimensional fields without considering boundary and end effects, the machine consist of two smooth coaxial cylinders made of magnetic material, the cylinders are separated by the air gap, the conductors of infinitely small cross-section are located in the air gap. Harmonics of other origin were not involved. See Appendix for details. Since the derivations as well as the conclusions that can be drawn from the model are based on the fundamental laws of electrical engineering, they do not require validation.

2. Calculation of Synchronous Parasitic Torques and Radial Magnetic Forces

Order of harmonics by definition see Appendix:

$$\nu_a = 2mg_1 + 1; \quad \mu_a = e\, Z_2/p + \nu_a = e \cdot 2mq_2' + \nu_a \tag{1a}$$

$$\nu_b = 2mg_2 + 1 \tag{1b}$$

where

ν_a, ν_b, μ_a harmonic order numbers stator, rotor
e, g_1, g_2 different integers: 0, ±1, ±2, ±3 …
m number of phases
Z_1, Z_2 slot numbers stator, rotor
2p pole number
q_1, q_2' relative slot numbers: slot number per phase per pole, stator / rotor

The formula for the calculation of the synchronous torque, which was first derived by us ([13] (13)):

$$\frac{M_{synchronous}}{M_{breakdown}} = \frac{X_m}{X_s} \cdot 2\sum \frac{\xi_{1\nu_a}\xi_{1\nu_b}}{\mu_a}\eta^2_{2\nu_a}\frac{1}{\xi_1^2} \tag{2}$$

where

$$\eta_{2\nu} = \frac{\sin\nu\frac{p\pi}{Z_2}}{\nu\frac{p\pi}{Z_2}} = \frac{\sin\nu\frac{\pi}{2mq_2'}}{\nu\frac{\pi}{2mq_2'}}$$

M torque
X_m, X_s reactance, magnetizing, leakage
$\xi_{1\nu a}$, $\xi_{1\nu b}$, ξ_1 winding factor of harmonics, of fundamental harmonic
$\eta_{2\nu}^2$ Jordan’s coupling factor [19] (258b)

The formulas for calculation of radial magnetic forces also first derived by us [13] (25), (26):
Radial magnetic force wave r=0, 2, 3,…

$$f = \frac{W_m}{(V_{airgap}/2)}C \quad [\mathrm{N/m^2}] \tag{3a}$$

One sided magnetic pull

$$F_{r=1} = \frac{W_m}{\delta'}C \quad [\mathrm{N}] \tag{3b}$$

where the proportionality factor C is

$$C = \frac{\xi_{1\nu_b}\xi_{1\nu_a}}{\nu_b\mu_a}\eta^2_{2\nu_a}\frac{1}{\xi_1^2} \tag{3c}$$

W_m magnetic energy of the machine
V volume of airgap
δ’ equivalent airgap

It is worth repeating our diagram developed in [14], which shows the change of the core element of the calculation $\eta_{2\nu}^2$ in a very visual way. It shows how much the rotor responds to a stator harmonic. A zero or very low value indicates that the rotor does not respond to that harmonic.

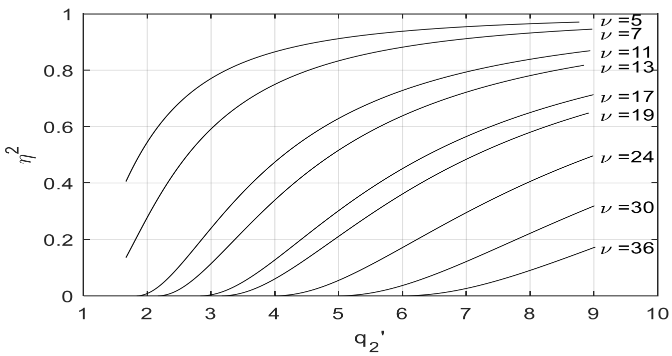

Figure 1: Representation of the value of $\eta_{2\nu}^2$ as a function of q_2' with ν as a parameter, for 3-phase machines. Higher odd harmonics are replaced mathematically by adjacent (actually with 3-phase not existing) even harmonics only for better transparency.

3. Synchronous Parasitic Torques and Radial Magnetic Forces in Three-Phase Machines

The rotor slot numbers resulting in a significant synchronous parasitic torque in the three-phase machine were presented in [16] Table III.A and those creating force waves of order r = 0 – 4 in [16] Table IV. Although the tables were elaborated independently of each other, still they clearly show the correlation between the parasitic torques and the force waves with order number r=0. These tables are not repeated here due to space limitations.

Now, using the new formulas a number of tables have been created like Table 1. for a wide range of relative rotor slot numbers. In order to make it easier for the Reader to follow, a simplification and approximation $\xi_{1a}\cdot\eta_{2v}^2/\xi_1^2\approx 1$ is considered, since only the quantities produced by the fundamental harmonic are considered for the moment: ν_a =1; this is generally a legitimate approximation [14] Figure 4. A machine with typical characteristics of X_m/X_s=15 was taken as basis with X_m=3 p.u., X_s=0.2 p.u., with this values M_{break} is usually ≈ 2 p.u. . The oscillating torque was calculated by the synchronous torque divided by the square of the value $I_{starting}/I_{rated}$ = 5.

Thus the formulas applied:

$$\frac{M_{synchronous}}{M_{breakdown}} = 15 \cdot 2 \frac{\xi_{1\nu_b}}{\mu_a} \tag{4a}$$

$$C = \frac{\xi_{1\nu_b}}{\mu_a^2} \tag{4b}$$

Then these tables were summarized in a *single table* in Table 2. representing the *point of subject development*. For space limitation only those relative rotor slot numbers were included in Table 2. which are possible in a 4 pole machine excl. odd rotor slot numbers.

After that, it was easy to enter the evaluation of the individual rotor slot numbers. The obviously prohibited slot numbers were marked in red, the recommended ones in green, and the slot numbers that are permitted under certain conditions (suitably high torque characteristics of the machine during start-up) in yellow. Torques in brake range were not included in the evaluation. The limit of the classification was defined as the synchronous parasitic torque is approx. 0.25times breakdown torque. This means 1% for oscillating torques. In the case of a high power motor, this limit must be reduced. While the torques are real values, the C factor is only a proportionality factor, the actual force obviously depends from the size.

It is emphasized that the table includes each relative rotor slot number possible at 4 pole and due to working with relative slot numbers, relative synchronous torques, relative proportionality factors and data of a typical motor the table is valid for any motor, with any pole number (consider the relation 2mq=Z/p). The entire system could not have been built if the parasitic torque was not related to the breakdown torque at the beginning, when the formula itself was derived.

For a certain q_2' listed in Table 2 the machine's behavior and data are *numerically identical* on any pole number; this refers for the evaluation as well. Therefore, Table 2. *comprehensively* covers the entire range of three-phase asynchronous machines as a whole. At pole numbers higher than 4, all that happens is that

Table 1: Harmonics, Oscillating Torques and Radial Magnetic Force Waves of a Three-Phase Machine for a Relative Rotor Slot Number q_2'= 2 1/3.

m=		**3**	ν_b			q_1			ξ_b			Synchronous Torque / Breakdown Torque			Oscillating Torque / Breakdown Torque %			C proportionality factor for radial magnetic force wave density f[N/m²] *10000			C proportionality factor for radial magnetic force wave density f[N/m²] *10000 in dB		
q_2	p/r	e	$\nu_b=-\mu_a$	$\nu_b=\mu_a$	operation	q_1=2	q_1=3	q_1=4	q_1=2	q_1=3	q_1=4	q_1=2	q_1=3	q_1=4	q_1=2	q_1=3	q_1=4	q_1=2	q_1=3	q_1=4	q_1=2	q_1=3	q_1=4
2	**1/3**	-1	13		motoric	ξ_{slot}			-0,966	0,218	0,126	-2,23	0,50	0,29	-8,92	2,01	1,16	-57,16	12,87	7,46	17,57	11,10	8,73
		2	-29		break				0,259	-0,177	-0,205	-0,27	0,18	0,21	-1,07	0,73	0,85	3,08	-2,11	-2,44	4,88	3,24	3,88
		-3		-41	standstill				-0,259	0,218	-0,158	≈0	≈0	≈0	≈0	≈0	≈0	≈0	≈0	≈0	≈0	≈0	≈0
		3		43					0,259	-0,177	0,205												
		-4	55		motoric		ξ_{slot}		-0,259	0,960	-0,158	-0,14	0,52	-0,09	-0,56	2,09	-0,34	-0,86	3,17	-0,52	-0,68	5,01	-2,83
		5	-71		break	ξ_{slot}	ξ_{slot}	ξ_{slot}	0,966	0,960	-0,958	-0,41	-0,41	0,40	-1,63	-1,62	1,62	1,92	1,90	-1,90	2,82	2,80	2,79
		-6		-83	standstill	ξ_{slot}			-0,966	-0,177	0,126	0,35	≈0	≈0	1,40	≈0	≈0	-1,40	≈0	≈0	1,47	≈0	≈0
		6		85					-0,966	0,218	-0,126	-0,34			-1,36			-1,34			1,26		
		-9		-125	standstill		ξ_{slot}		0,259	0,960	-0,205	≈0	-0,23	≈0	≈0	-0,92	≈0	≈0	0,61	≈0	≈0	-2,12	≈0
		9		127					-0,259	0,960	0,158		0,23			0,91			0,60			-2,25	
		-12		-167	standstill	ξ_{slot}		ξ_{slot}	0,966	0,218	-0,958	-0,17	≈0	0,17	-0,69	≈0	0,69	0,35	≈0	-0,34	-4,60	≈0	-4,64
		12		169					0,966	-0,177	-0,958	0,17		-0,17	0,69		-0,68	0,34		-0,34	-4,71		-4,75

further relative rotor slot numbers like “+1/9q”, “+1/12q”, “1/4q” etc. become possible, the table is easy to supplement with them by creating further tables like Table 1. with a little investment of time; at high pole numbers and/or high phase numbers, odd slot numbers do not necessarily have to be ruled out.

Table 2: Largest Synchronous Torques, Oscillating Torques and the Largest Proportionality Factors of the Radial Magnetic Force Waves for a Three-Phase Machine

Relative rotor slot number q_2'	Operation	Synchronous Torque / Breakdown Torque			Oscillating Torque / Breakdown Torque %			C proportionality factor for radial magnetic force wave density [N/m^2]·10000		
		q_1=2	q_1=3	q_1=4	q_1=2	q_1=3	q_1=4	q_1=2	q_1=3	q_1=4
q_1-2+**2/3**	motor		1,52	0,15		6,06	0,61		26,59	1,64
	brake	x	0,48	0,28	x	1,93	1,11	x	14,66	5,45
	standstill		0,64	1,20		2,56	4,80		2,37	8,33
q_1-2+**5/6**	motor		0,26	0,17		1,06	0,68		0,96	0,46
	brake	x	0,32	0,11	x	1,29	0,43	x	4,11	1,03
	standstill		0,29	0,00		1,16	0,00		0,49	0,00
q_1-1+**1/6**	motor	2,23	0,21	0,10	8,92	0,85	0,41	57,16	2,84	0,92
	brake	0,41	0,54	0,08	1,63	2,17	0,32	3,08	3,42	0,35
	standstill	0,68	0,24	0,00	2,72	0,96	0,00	2,74	0,35	0,00
q_1-1+**1/3**	motor	1,11	0,52	0,32	4,44	2,09	1,30	52,8	12,87	5,69
	brake	0,46	0,41	0,12	1,83	1,62	0,46	8,96	2,11	0,94
	standstill	2,42	0,46	0,48	9,68	1,84	1,92	33,70	1,21	1,33
q_1+**2/3**	motor	0,25	0,12	0,09	1,00	0,49	0,34	2,69	0,96	0,52
	brake	0,46	0,32	0,21	1,83	1,29	0,85	8,96	4,11	2,44
	standstill	1,21	0,22	0,34	4,84	0,88	1,36	8,39	0,30	0,68
q_1+**5/6**	motor	0,17	0,32	0,05	0,69	1,27	0,21	0,58	1,16	0,16
	brake	0,83	0,11	0,06	3,31	0,45	0,26	7,89	0,80	0,36
	standstill	0,28	0,00	0,00	1,12	0,00	0,00	0,46	0,00	0,00

Table 2. does not include the magnitude of the asynchronous parasitic torque. Consider the formula also derived by us first [15] (19)

$$\frac{M_{break\,\nu}}{M_{break}} = \frac{X_m}{X_s}\frac{\xi_\nu^2}{\xi_1^2}\frac{\eta_{2\nu}^2}{\nu} \qquad (5)$$

Considering Figure 1. it is clear that if $Z_2<Z_1$, or if $Z_2>Z_1$, but according to practice is not much larger, then there is no need to deal with asynchronous parasitic torque because $\eta_{2\nu}^2$ is zero or very small. In extreme cases, (5) must be used, but we will not deal with such a rare case here. Nevertheless, machines with q_1=2 and q_2'>q_1 might suffer because of this phenomenon.

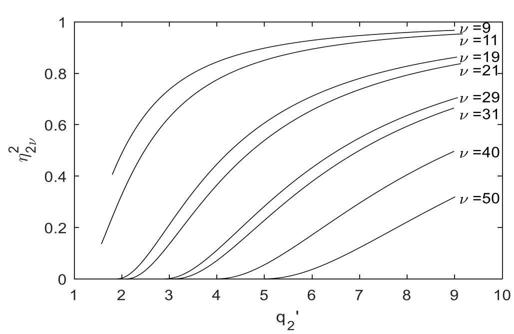

Figure 2: Representation of the value of $\eta_{2\nu}^2$ as a function of q_2' with ν as a parameter, for 5-phase machines. Higher odd harmonics are replaced mathematically by adjacent (actually with 5-phase not existing) even harmonics only for better transparency.

4. Synchronous Parasitic Torques and Radial Magnetic Forces in Five-Phase Machines

The study of this paragraph is started with Figure 2. showing the value of $\eta_{2\nu}^2$ as a function of q_2' with ν as a parameter defined for 5-phase machines, just for reference. It is not surprising that the figure is very similar to Figure 1.

The 5-phase machines are powered almost exclusively via an inverter, where the synchronous parasitic torque is of no importance. However, it is important that all of these appear as oscillating torques in operation and as a consequence radial magnetic forces appear at the same time influencing vibration and noise on the same way as in net-supplied machines. Therefore motors for inverter shall be designed that means the rotor slot number must be selected as if it were a mains powered motor with constant (rated) speed.

4.1. Slot numbers creating considerable synchronous parasitic torques

It was seen that, especially in machines with chording, the effect of higher ν_a harmonics is small. And in the case of 5-phase machines, since the first harmonics are already -9 and 11 instead of -5 and 7, their effect will be negligible even in non-chorded machines. Therefore, rotor harmonics created by the fundamental stator harmonic will be considered only.

$$\nu_a = 10g_1 + 1 \quad \rightarrow \quad \nu_a = 1, \qquad (6a)$$

$$\nu_b = 10g_2 + 1 \quad \rightarrow \quad \nu_b = 1, -9, 11, 19, 21, -29, 31 \ldots \qquad (6b)$$

$$\mu_a = e \cdot Z_2/p + 1 = e \cdot 2mq_2' + 1 \qquad (6c)$$

Compared with the harmonics of a three-phase machine, it can be seen that higher order numbers occur in the ratio of 5/3 in the five-phase machine. Therefore, it is expected that both its oscillating torque and radial magnetic force waves will be smaller; therefore it will be easier to find an ideal rotor slot number between the fewer prohibited ones.

In Table 3., the rotor slot numbers for the pole numbers 2p = 4 – 8, which generate significant synchronous torques are specified.

The tables show that the most critical slot numbers are: q_2'=integer, q_2'=q_2±1/5 obtained by e=±1.

4.2. Slot numbers creating considerable radial magnetic forces

The rotor slot numbers creating considerable (that means low order) radial magnetic forces are given in Table 4. As seen those ones given in 4.1 create significant radial magnetic forces of order number r=0. The slot numbers that appear in the same row result in radial magnetic forces with the same magnitude, due to $\xi_{1\nu b}$ is identical and differences in μ_a and ν_b with r= 1 – 4 are negligible.

The natural frequencies of the actual machine can be checked with the reported frequencies. These frequencies were calculated now with 50 Hz “rated” frequency.

In Table IV. of [16] referring to 3-phase 4 pole machines, the rotor slot ranges belonging to a certain q_1 “touched” each other that means there was no rotor slot number that does not generate radial magnetic forces with low order number. Here, however, *there are* rotor slot numbers between the ranges belonging to two adjacent q_1 without this harmful effect. At large machines, however, where r=5, 6 matter, the table must be supplemented by those columns, then each rotor slot number will again generate low-order radial magnetic force wave.

Table 3: Rotor Slot Numbers that Create Considerable Synchronous Parasitic Torques for a Five-Phase Machine

m	5			p=	2				
				e=		e=		e=	
				-1	1	-2	2	-3	3
q_1	Z_1	μ_a	μ_a						
		$\mu_a = \nu_b$	$\mu_a = -\nu_b$						
1	20	-9		**20**		**10**			
			9		**16**		**8**		
		11			**20**		**10**		
			-11	**24**		**12**		**8**	
2	40	-19		**40**		**20**			
			19		**36**		**18**		**12**
		21			**40**		**20**		
			-21	**44**		**22**			
3	60	-29		**60**		**30**		**20**	
			29		**56**		**28**		
		31			**60**		**30**		**20**
			-31	**64**		**32**			
4	80	-39		**80**		**40**			
			39		**76**		**38**		
		41			**80**		**40**		
		-41	-41	**84**		**42**		**28**	

m	5			p=	3				
				e=		e=		e=	
				-1	1	-2	2	-3	3
q_1	Z_1	μ_a	μ_a						
		$\mu_a = \nu_b$	$\mu_a = -\nu_b$						
1	30	-9		**30**		**15**		**10**	
			9		**24**		**12**		**8**
		11			**30**		**15**		**10**
			-11	**36**		**18**		**12**	
2	60	-19		**60**		**30**		**20**	
			19		**54**		**27**		**18**
		21			**60**		**30**		**20**
			-21	**66**		**33**		**22**	
3	90	-29		**90**		**45**		**30**	
			29		**84**		**42**		**28**
		31			**90**		**45**		**30**
			-31	**96**		**48**		**32**	
4	120	-39		**120**		**60**		**40**	
			39		**114**		**57**		**38**
		41			**120**		**60**		**40**
			-41	**126**		**63**		**42**	

m	5			p=	4				
				e=		e=		e=	
				-1	1	-2	2	-3	3
q_1	Z_1	μ_a	μ_a						
		$\mu_a = \nu_b$	$\mu_a = -\nu_b$						
1	40	-9		**40**		**20**			
			9		**32**		**16**		
		11			**40**		**20**		
			-11	**48**		**24**		**16**	
2	80	-19		**80**		**40**			
			19		**72**		**36**		**24**
		21			**80**		**40**		
			-21	**88**		**44**			
3	120	-29		**120**		**60**		**40**	
			29		**112**		**56**		
		31			**120**		**60**		**40**
			-31	**128**		**64**			
4	160	-39		**160**		**80**			
			39		**152**		**76**		
		41			**160**		**80**		
			-41	**168**		**84**		**56**	

Table 4: Force Wave Order Numbers Created by the Given Rotor Slot Numbers and the Frequencies of the Wave, for Five Phases; Supply Frequency 50 Hz

m=	5			p=	2								
				r=	**0**	r=	**1**	r=	**2**	r=	**3**	r=	**4**
				+	-	+	-	+	-	+	-	+	-
q_1	Z_1	ν_b	ν'_b										
2	40	-19	-38	**36**	**40**	**37**	**41**	**38**	**42**	**39**	**43**	**40**	**44**
		frequency Hz		1000	1000	1025	1025	1050	1050	1075	1075	1100	1100
			e=±2	**18**	**20**			**19**	**21**			**20**	**22**
			e=±3	**12**					**13**				
		-19	-38	**36**	**40**	**35**	**39**	**34**	**38**	**33**	**37**	**32**	**36**
		frequency Hz		1000	1000	975	975	950	950	925	925	900	900
			e=±2	**18**	**20**			**17**	**19**			**16**	**18**
			e=±3	**12**			**13**			**11**			**12**
		21	42	**44**	**40**	**45**	**41**	**46**	**42**	**47**	**43**	**48**	**44**
		frequency Hz		1000	1000	1025	1025	1050	1050	1075	1075	1100	1100
			e=±2	**22**	**20**			**23**	**21**			**24**	**22**
			e=±3			**15**			**14**			**16**	
		21	42	**44**	**40**	**43**	**39**	**42**	**38**	**41**	**37**	**40**	**36**
		frequency Hz		1000	1000	975	975	950	950	925	925	900	900
			e=±2	**22**	**20**			**21**	**19**			**20**	**18**
			e=±3				**13**	**14**					**12**
3	60	-29	-58	**56**	**60**	**57**	**61**	**58**	**62**	**59**	**63**	**60**	**64**
		frequency Hz		1500	1500	1525	1525	1550	1550	1575	1575	1600	1600
			e=±2	**28**	**30**			**29**	**31**			**30**	**32**
			e=±3		**20**	**19**					**21**	**20**	
		-29	-58	**56**	**60**	**55**	**59**	**54**	**58**	**53**	**57**	**52**	**56**
		frequency Hz		1500	1500	1475	1475	1450	1450	1425	1425	1400	1400
			e=±2	**28**	**30**			**27**	**29**			**26**	**28**
			e=±3		**20**			**18**					
		31	62	**64**	**60**	**65**	**61**	**66**	**62**	**67**	**63**	**68**	**64**
		frequency Hz		1500	1500	1525	1525	1550	1550	1575	1575	1600	1600
			e=±2	**32**	**30**			**33**	**31**			**34**	**32**
			e=±3		**20**			**22**			**21**		
		31	62	**64**	**60**	**63**	**59**	**62**	**58**	**61**	**57**	**60**	**56**
		frequency Hz		1500	1500	1475	1475	1450	1450	1425	1425	1400	1400
			e=±2	**32**	**30**			**31**	**29**			**30**	**28**
			e=±3		**20**	**21**					**19**	**20**	
4	80	-39	-78	**76**	**80**	**77**	**81**	**78**	**82**	**79**	**83**	**80**	**84**
		frequency Hz		2000	2000	2025	2025	2050	2050	2075	2075	2100	2100
			e=±2	**38**	**40**			**39**	**41**			**40**	**42**
			e=±3										
		-39	-78	**76**	**80**	**75**	**79**	**74**	**78**	**73**	**77**	**72**	**76**
		frequency Hz		2000	2000	1975	1975	1950	1950	1925	1925	1900	1900
			e=±2	**38**	**40**			**37**	**39**			**36**	**38**
			e=±3			**25**			**26**			**24**	
		41	82	**84**	**80**	**85**	**81**	**86**	**82**	**87**	**83**	**88**	**84**
		frequency Hz		2000	2000	2025	2025	2050	2050	2075	2075	2100	2100
			e=±2	**42**	**40**			**43**	**41**			**44**	**42**
			e=±3				**27**						
		41	82	**84**	**80**	**83**	**79**	**82**	**78**	**81**	**77**	**80**	**76**
		frek. Hz		2000	2000	1975	1975	1950	1950	1925	1925	1900	1900
			e=2	**42**	**40**			**41**	**39**			**40**	**38**
			e=3						**26**				

Still due to space limitations, the table is published here only for 4 poles and only for relative stator slot numbers q_1=2, 3, 4. However, this will *not* be sufficient for a complete analysis, it is necessary to reveal all the cases where a certain rotor slot number to be examined and any integer multiple of it appears anywhere, therefore the missing table parts q_1=1, 5... are also necessary; nevertheless they can be easily created based on the sample with

Table 5: Harmonics, Oscillating Torques and Radial Magnetic Force Waves of a Five-Phase Machine for a Relative Rotor Slot Number q_2'= 2 1/5

m		5	ν_b			q_1			ξ_b			Synchronous Torque / Breakdown Torque			Oscillating Torque / Breakdown Torque %			C proportionality factor for radial magnetic force wave density f[N/m^2] *10000		
q_2	p/r	e	$\nu_b=-\mu_a$	$\nu_b=\mu_a$	operation	q_1=2	q_1=3	q_1=4	q_1=2	q_1=3	q_1=4	q_1=2	q_1=3	q_1=4	q_1=2	q_1=3	q_1=4	q_1=2	q_1=3	q_1=4
2	**1/5**	-1	21		motoric	ξ_{slot}			-0,988	0,127	0,077	-1,411	0,182	0,111	-5,64	0,73	0,44	13,50	4,60	2,45
		1	n.a.	n.a.																
		-4	n.a.	n.a.																
		4	-89		break		ξ_{slot}		0,156	0,985	0,119	-0,053	-0,332	-0,040	-0,21	-1,33	-0,16	-7,04	0,95	-8,23
		-5		-109	standstill				-0,156	-0,113	0,102	0,043	0,031	-0,028	0,00	0	0	0	0	0
		5		111					0,156	0,127	-0,119	0,042	0,034	-0,032						
		-10		-219	standstill	ξslot			-0,988	0,127	0,077	0,135	-0,017	-0,011	0,54	0	0	-6,86	0	0
		10		221					-0,988	-0,113	-0,077	-0,134	-0,015	-0,011	-0,54			-6,94		

a little investment of time, the same was described and already published for 3 phases in [16].

In accordance with Table 1, we also provide in Table 5. below an example for 5-phase on how to calculate the order number of harmonics generated by a certain relative rotor slot number and the values created by it. We stayed with the reactance ratio value of (2) according to section 3, even though it differs from that of a three-phase machine [3], [7]-[9]. On the one hand, the linear conversion to the actual reactance ratio of the machine to be investigated does not cause any problem, on the other hand, the table shows clearly anyway which harmonics $\xi_{1\nu b}=\xi_1$ will apply for, i.e. are slot harmonics, being therefore dangerous and should be avoided. Equation (4b), as a relative number, remains still valid here.

We are not dealing with the speed range in which the parasitic synchronous torque occurs at 5-phase machines: we need them only for calculation of the oscillating torque; here always the largest of them was chosen. The largest value of the proportionality factor C does not necessarily belong to the largest value of the oscillating torque, but - if several torques of a similar magnitude are generated – it belongs to the one produced by the *lowest* order harmonic ν_b. That phenomenon is an important hint regarding noise, because it means that the noise is caused by only few, high noise component created by low order harmonics.

Table 6: Largest Oscillating Torques and the Largest Proportionality Factors of the Radial Magnetic Force Waves for Five-Phase Machines

Relative rotor slot number q_2'	Oscillating Torque / Breakdown Torque %			C proportionality factor for radial magnetic force wave density f [N/m^2] ·10000		
	q_1=2	q_1=3	q_1=4	q_1=2	q_1=3	q_1=4
q_1-2+**9/10**	x	0,78	0,32	x	0,84	0,22
q_1-1+**1/10**	5,64	1,13	0,15	22,40	0,70	0,21
q_1+**9/10**	4,09	0,35	0,19	2,84	0,18	0,08
q_1-2+**4/5**	x	2,63	0,42	x	3,12	1,21
q_1-1+**1/5**	3,95	1,33	1,47	12,93	2,89	1,24
q_1+**4/5**	1,69	0,78	0,98	1,86	0,84	0,5
q_1-2+**7/10**	x	0,49	0,73	x	0,27	0,38
q_1-1+**3/10**	1,50	1,30	0,59	1,58	1,19	0,25
q_1+**7/10**	0,74	0,79	0,42	0,38	0,44	0,12
q_1-2+**3/5**	x	3,81	1,5	x	10,25	1,58
q_1-1+**2/5**	2,89	0,98	0,49	5,88	0,45	0,21
q_1+**3/5**	1,5	1,31	0,13	1,58	0,61	0,12
q_1**±1/2**	≈ 6	≈ 2	≈ 0,5	≈ 10	≈ 0,5	≈ 0,1

A number of tables were created like Table 5. for a wide range of relative rotor slot numbers and then summarized in a single core table in Table 6., again *a point of subject development.*

Again for space limitation only those relative rotor slot numbers were included in Table 6. which are possible in a 4 pole machine. Odd rotor slot numbers remain still disregarded although the situation is not as strong as for 3-phase machines. As it can be seen - and in accordance with the expectations - the 5-phase machines are superior to 3-phase machines with regard to oscillating torques and the radial magnetic force density. However, unlike three-phase machines, we do not introduce here direct evaluation. In the case of a three-phase, mains-fed motor, the start-up, i.e. the *operation* itself becomes questionable or impossible in the case of an inadequate rotor slot number. However, there is no such danger in the case of a 5-phase machine powered by an inverter. In our opinion, however, a limit still should be introduced here as well, let it be again the value of 1% of the oscillating torque. After all, the 5-phase machine is used expressly in the hope of calm operation and less noise as a result of the smaller C proportionality factors. Should the data of the machine creep up into the range of three-phase machines just because the wrong rotor slot number it has no sense anymore to assume the excess costs of a non-standard motor plus an inverter.

As expected corresponding to Table 3, Table 6. shows that the "1/5" – "4/5" machines are the least favorable but "2/5 – 3/5" slot numbers are also better to avoid.

5. Further General Considerations, Selection of Rotor Slot Number

Write down again the rotor harmonics for ν_a=1 expressed by relative rotor slot number q_2':

$$\mu_a = e \cdot 2mq_2' + 1 \tag{7}$$

where as before

- e ±1, ±2, ±3…, integers
- $q_2'=q_2+p/r$ with meaning that any relative rotor slot number can always be expressed by an improper fraction
- q_2 positive integer
- p, r positive integers and p≤r.

Substituting

$$\mu_a = e \cdot 2m(q_2 + \frac{p}{r}) + 1 = e \cdot 2mq_2 + e \cdot 2m\frac{p}{r} + 1 \tag{8}$$

A necessary (but not sufficient) condition for the generation of synchronous parasitic torque is that μ_a is odd integer and in (8) the sum of the first two terms is even that means the second term is also even.

If we select the rotor slot number such that r=m or p/r=1 ("integer" rotor slot number), then this condition is already satisfied with the lowest value of e: e=±1. This means that with such a selection, the order number of synchronous torques will be as small as possible, and the synchronous torque itself will probably be as high as possible. The term "probably" refers to the fact that the torque still depends on whether $\xi_{1vb} = \xi_1$ or not see (3c), (4a), (4b). The final condition is that μ_a shall be identical to a stator harmonic (1b), (6b). This is fulfilled as above if q_2' = integer or if $q_2' = q_2 \pm 1/m$.

These (dangerous) rotor slot numbers are for 3-phase

$$q_2' = q_2, \quad q_2' = q_2 + 1/3, \quad q_2' = q_2 + 2/3 ,$$

for 5-phase

$$q_2' = q_2, \quad q_2' = q_2 + 1/5, \quad q_2' = q_2 - 1/5$$

and, regardless of the phase number, even though with e=±2, still the

$$q_2' = q_2 + 1/2$$

slot numbers.

Staying with the most common practical case that r=m, the maximum torques are generated alternately in braking mode and in motoric operation in three-phase by substitution e=±1 and e=±2, and always in standstill by substitution e=±3, then the same pattern is repeated with further <u>e</u> values [17]. The same substitutions e=±1, e=±4 and e=±5 cause the same effect in 5-phase, respectively. In industrial motor, a high torque generated during braking is not a problem. In the case of low-noise machines, no matter net supplied or via inverter, however, where these always produce significant force waves with order number r = 0 and significant oscillating torques, these slot numbers should be avoided at all costs.

Further considerations can be found in [17] Chapter III. G.

5.1. Influence of the phase number

The flux distribution images and animations created by FEM simulations do not show anything about the number of phases of the winding. So, the FEM simulation hides the fact that the influence of the phase number is also included in the circumferential change of the MMF, and that gives the MMF an additional specific periodicity, which is very important from the point of view of the present investigation. This is the case if $q_1>1$, it will be most visible with q_1 integer, full pitch winding. Instead of MMF of each slot differing in phase from the previous one by one slot pitch, like in the rotor, groups of slots consisting of q_1 slots with the same phase follow one after the other. This is the phenomenon which creates a periodicity according to the phase number. In the formulas: in the case of 3 phases, every third odd MMF harmonic is missing or the order numbers of the MMF harmonics are adjacent to every third even number and in the case of 5 phases, to every fifth even number. The rotor slot number must, therefore, be as far away from that periodicity as possible or the torque still created be as low as possible. Periodicity according to the phase number is created when the second term of (8) is an even integer multiple of the phase number; if it is not possible to avoid then as higher multiple as better ($q_2'=q_1$ and q_2'=integer are the worst cases). In such cases only standstill synchronous parasitic torques are created. Further analysis is required for achieving general results.

6. Design rules

The question is examined now in its entirety, using both our previous and current results. The wording is mainly related to 3-phase machines. The recommendations will not apply to individual slot numbers, which would be the approach we do not want, but to bands: recommended, to be avoided and prohibited bands will be defined. These bands sometimes slide into each other, they sometimes represent conflicting recommendations. The wording is corresponding to (8) especially to second term of it. The sequence of considerations shall be:

- the basic law is: it is *the p/r proper fractional part of the relative rotor slot number which determines the behavior of the squirrel cage asynchronous machine.* The denominator <u>r</u> determines the dangerousness of the rotor slot number, and the numerator determines the speed range in which the parasitic torque occurs when r=m. Therefore, during the entire investigation, above and below, the rotor slot number is expressed and named in the form of p/r without q_2.
- referring to (4a) and (4b) the lowest possible value of μ_a must be as high as possible. Referring to (8), therefore, the lowest possible value of <u>e</u> must be as high as possible; consequently *the value of <u>r</u> should be also as high as possible, this is the goal regarding the suitable rotor slot number.* As a default, *only standstill* synchronous parasitic torques are created.
- referring to (8), with p/r=0 (equal to p/r=1) and r=2 (p/r=1/2), however, still unacceptably high synchronous parasitic torques are created therefore they are *forbidden*
- slot numbers that create radial magnetic forces with order number r=1 are prohibited, even if they are created by a ξ_{1vb} non-slotharmonic; *these are always Z_2= odd slot numbers therefore they are forbidden;* even though these would have the advantage of creating only a small, practically negligible synchronous parasitic torque.
- in the case of r=3, 4, 5...acc. to (8), the magnitude of the highest synchronous parasitic torque is decreasing, they are acceptable, in fact, they are particularly low; further: e<r does not create synchronous parasitic torque. *Therefore, these slot numbers are expressly recommended, in fact, they should be selected primarily, but with condition r≠m.* The lowest pole number is 2p=8 where the condition r≠m or r≠integer·m can be met: the slot numbers *recommended* are: 1/4, 3/4.
- additional conditions apply, however, to slot numbers that <u>r</u> in (8) is equal to the phase number or multiples thereof: r=m, r=2m, r=3m etc. Per definition: r=m is possible at any pole number. As a matter of fact, in the case of 2p=2, 4, and 6, there is no other option (with the exception of odd slot numbers, which are otherwise prohibited especially at these low pole numbers due to dangerous radial forces of r=1). These slot numbers, in addition to standstill synchronous parasitic torques, create torques *also in rotation*. As a rule for r=m: substitution e=1 and e=-2 or e=-1 and e=2 create synchronous parasitic torques always in rotation; e=±3 (in pair) create in standstill.

Then this pattern repeats itself. Those with e=±1 are large, m times larger than the standstill torques created by e=m=3. Therefore, these slot numbers are to be selected only if there is no other choice, if the slot numbers r≠m are not possible. It should be noted that, despite of this, these are the most frequently used rotor slot numbers, even when there are acc. to the author more favorable options (e.g. in 4 pole).

- Possible slot numbers remaining for 2 pole: 1/3, 2/3, for 4 pole: 1/3, 2/3, 1/6, 5/6; for 6 pole: 1/3, 2/3, 1/9, 2/9, 4/9, 5/9, 7/9, 8/9.
- slot numbers creating the highest torque in the motor range with e=±1 therefore are to be *avoided*: 1/3, 1/6, 1/9, 4/9, 7/9, 1/12, 7/12.
- slot numbers creating the highest torque in the brake range therefore they are *recommended*: 2/3, 5/6, 2/9, 5/9, 8/9, 5/12, 11/12.
- the band q_1-1/2≤q_2'≤q_1+1/2 is *prohibited*, since then the low order radial magnetic forces are created by the $\xi_{1\nu b}$ slotharmonic
- band q_1-1<q_2'<q_1-1+1/2 is rather *not recommended* because the most of the slot numbers (except 2/9, 5/12 …) falling into this band create the highest torque in the motor range
- the remaining bands that may be *recommended*: q_1-2+2/3≤q_2'<q_1-1 and q_1+2/3≤q_2'<q_1+1 bands because the slot numbers falling into this band create the highest torque in the brake range
- the rule regarding Z_2=odd slot numbers is not so strict for high pole numbers. In that case, *there are* odd slot numbers that do not create r=1 force wave, and some of them that neither r=1 nor r=3 force wave: the former might be used, the latter are *expressly recommended* slot numbers. The rule q_1-1/2≤q_2'≤q_1+1/2 is also not so strict for high pole numbers and high phase numbers.
- there are special situations where further considerations apply:
- if the stator winding also creates even harmonics, such as some pole changing winding, q_2'<q_1 must be selected to avoid the high asynchronous parasitic torque created by the stator harmonic ν_a=4 in the motor range
- q_2'>q_1 should be selected to achieve lower synchronous parasitic torques and radial magnetic forces (noise)
- q_2'<1.25q_1 should be selected to avoid troublesome asynchronous parasitic torques
- finally: if the actual ν_b happens to be a slot harmonic with $\xi_{1\nu b}=\xi_1$ winding factor, the sequence of selection established above is changed: the concerned rotor slot number, which otherwise would belong to the range of recommended slot numbers, must be transferred to the prohibited ones
- since there is no rotor slot number that does not create synchronous parasitic torque, the design starts with checking the stator natural frequency for force wave order r=0.

7. Behavior of synchronous parasitic torques during run-up and at operating speed

As already found in [20] and cited in [15], the asynchronous machine with a squirrel cage rotor is to be modeled with small asynchronous machines ([15] Figure 2.) and *small synchronous machines* in shaft connection with the main motor in order to take into account the MMF harmonics. Now, for a deeper understanding of the behavior of synchronous parasitic torques, the small synchronous machines will be defined and their behavior will be studied.

First, we examine the operation of the asynchronous machine "taken hold" by the synchronous parasitic torque; by "taken hold" operation, we mean when the high synchronous parasitic torque does not allow the machine to run up, it either remains in standstill or runs on some s=(ν_b-1)/(ν_b+1) slip ([14] (11)). In this operating status, the usual equivalent circuit diagram of a synchronous machine can be used.

The corresponding element of the differential leakage will correspond to the synchronous reactance $X_d = X_m/\nu_b^2 \cdot \xi_b^2/\xi_1^2$, X_m is the fundamental harmonic magnetizing reactance of the asynchronous main machine. The mains voltage U corresponds to the voltage drop of the stator current on the investigated reactance element of the differential leakage:

$$U = I_1 X_d = I_1 \frac{X_m}{\nu_b^2} \frac{\xi_b^2}{\xi_1^2} \tag{9}$$

The U_p pole voltage corresponds to the voltage drop caused by the rotor current on a reactance of the same value - since $\nu_b=\pm\mu_a$ – as the element above:

$$U_p = I_2^{'} X_d = I_2^{'} \frac{X_m}{\mu_a^2} \frac{\xi_b^2}{\xi_1^2}; \quad I_2^{'} = -I_1 \frac{\xi_a}{\xi_b} \eta_2^2 \tag{10}$$

η_2^2 Jordan's coupling factor (2)

The reactance associated with the rotor voltage U_p *does not appear* even on the extended equivalent circuit diagram ([15] Figure 1); its value is the same as the reactance called X_d because, again, by definition $\nu_b=\pm\mu_a$; ν_b and ξ_b are the harmonic number and winding factor of the stator MMF harmonic involved in generating the synchronous parasitic torque; μ_a and ξ_a are the rotor harmonic number created by the stator harmonic ν_a and the stator harmonic winding factor ξ_a. The multiplier of I_1 takes into account that $\nu_a \neq \nu_b$, so ξ_a and ξ_b may differ. If we did not take this into account, contrary to our intention, we would calculate the quantities of the asynchronous parasitic torque.

Let's start our investigation with the simplest case, when $\nu_b=\mu_a$ (and both are positive) that is, the torque is generated in standstill.

Maximum power

$$P = \frac{U_1 U_2^{'}}{X_d} = I_1^2 \eta_2^2 \frac{X_m}{\nu_b^2} \frac{\xi_a \xi_b}{\xi_1^2} = I_1^2 \frac{\xi_a}{\xi_b} X_d \tag{11}$$

The torque of the small synchronous machine is created by the field of the harmonic ν_b with angular velocity ω_0/ν_b, therefore

$$M = \frac{Pm}{\omega_0 / \nu_b p} = \frac{m I_1^2 \eta_2^2}{\omega_0 / \nu_b p} \frac{X_m}{\nu_b^2} \frac{\xi_a \xi_b}{\xi_1^2} \tag{12}$$

where m phase number
p pole pair number
ω_0 electrical angular velocity of supply net

Fundamental harmonic breakdown torque and stator current now being equal to starting current

$$M_{break} = \frac{p}{\omega_0}\frac{mU^2}{2X_s} \; ; \qquad I = \frac{U}{X_s} \tag{13}$$

where U net voltage
X_s leakage reactance of (main) motor

Torque of the small synchronous motor

$$M = \frac{M_{synchr}}{M_{break}} = \frac{mU^2/X_s^2\eta_2^2}{\omega_0/\nu_b p}\frac{X_m}{\nu_b^2}\frac{\xi_a\xi_b}{\xi_1^2}\frac{\omega_0 2X_s}{pmU^2} = 2\frac{X_m}{X_s}\frac{\eta_2^2}{\nu_b}\frac{\xi_a\xi_b}{\xi_1^2} \tag{14}$$

Thus, our previously derived formula is verified here using another method of derivation, see [13] (13).

Let's examine what happens when the machine somehow starts, runs up and then runs at rated speed. Then the poles of the rotor rotate in space under the poles of the stator, meaning that this small machine is "out of synchronism".

The precise investigation cannot be performed on the basis of the extended equivalent circuit diagram either, for this we have to go back to the basic relationships of the unified electric machine theory. The simplest way to start is from Figure 3 ([21] p. 111), later we will also introduce three-phase space vector considerations.

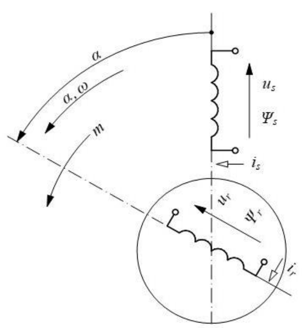

Figure 3: Two-winding basic slipring machine [21] p. 111.

Basic equations

$$u_s = i_s R_s + \frac{d\Psi_s}{dt} \qquad \Psi_s(\alpha,t) = l_s(\alpha) i_s + l_m(\alpha) i_r \tag{15}$$

$$u_r = i_r R_r + \frac{d\Psi_r}{dt} \qquad \Psi_r(\alpha,t) = l_m(\alpha) i_r + l_m(\alpha) i_r$$

$$\alpha(t) \qquad i(t)$$

$$u_s = i_s R_s + l_s\frac{di_s}{dt} + l_m\frac{di_r}{dt} + \omega i_s\frac{dl_s}{d\alpha} + \omega i_r\frac{dl_m}{d\alpha} \tag{16}$$

$$u_r = i_r R_r + l_r\frac{di_r}{dt} + l_m\frac{di_s}{dt} + \omega i_r\frac{dl_r}{d\alpha} + \omega i_s\frac{dl_m}{d\alpha}$$

$$m = \frac{1}{2}i_s^2\frac{dl_s}{d\alpha} + \frac{1}{2}i_r^2\frac{dl_r}{d\alpha} + i_s i_r\frac{dl_m}{d\alpha}$$

In (16), the first two induced voltages are the so-called transformer voltages, and the other two ones represent voltages due to rotation.

We are examining a cylindrical machine, therefore $dl_s/d\alpha=0$, $dl_r/d\alpha=0$. The dynamic phenomena that occur are not examined now ($d\omega/dt=0$). As an approximation and simplification, we also neglect resistances: this is a legitimate approach especially for low, dangerous ν_b harmonics. The perceptible synchronous parasitic torques generated in standstill are created by slot harmonics – this can be verified theoretically ([17], [18] Appendix) - therefore $\xi_b=\xi_a=\xi_1$.

First, the synchronous operation in standstill will be checked: $\omega=0$, $\alpha=$const. The equations

$$u_s = l_s\frac{di_s}{dt} + l_m\frac{di_r}{dt} \tag{17}$$

$$u_r = l_r\frac{di_r}{dt} + l_m\frac{di_s}{dt}$$

$$m = i_s i_r\frac{dl_m}{d\alpha}$$

Since the MMF of both the stator and the rotor exerts its effect on the same path, $l_s=l_r=l_m=L=1/\omega_0 \cdot X_m/\nu_b^2$ apart from the leakage to be interpreted for the actual small synchronous machine. The mutual inductance depends on the angle of the rotor position related to one rotor slot pitch: $l_m=L\cdot\cos\alpha=L\cdot e^{j\alpha}$.

Substituting and taking into account that the (excitation) currents are of 50 Hz and the system operates under forced current ($i_s=$const., $i_r=$const.), at a certain angle α of the rotor position, the equations in the own coordinate system of both the stator and the rotor

$$u_s = i_s j\frac{X_m}{\nu_b^2} + i_r\cos\alpha j\frac{X_m}{\nu_b^2} = j\frac{X_m}{\nu_b^2}(i_s + i_r e^{j\alpha}) \tag{18}$$

$$u_r = i_r j\frac{X_m}{\nu_b^2} + i_s\cos\alpha j\frac{X_m}{\nu_b^2} = j\frac{X_m}{\nu_b^2}(i_r + i_s e^{-j\alpha})$$

$$m = i_s i_r\frac{X_m}{\nu_b^2}(-\sin\alpha) = i_s i_r j\frac{X_m}{\nu_b^2}e^{j\alpha}$$

Equations of a synchronous machine are obtained. The maximum torque occurs at $\alpha=\pi/2$, in which case also i_s and i_r are perpendicular to each other, consequently, as in the usual vector diagram of a synchronous machine, u_s and u_r that means U and U_p are also perpendicular to each other. Omitting the derivation, our original formulas ([13] (13)) can also be derived in this way.

The examination of the synchronous torque during rotation will be omitted, the result is the same.

Next, the machine is investigated during start-up. This requires a more precise formulation of the task, since here in addition to transformer voltages, so called rotational voltages also appear. Although these are also included in the equations of the basic machine, we still choose the full three-phase vector theory method; within this, the common coordinate system attached to the stator is expedient [21] (10-19).

The basis of our calculation is [19] (309), according to which the rotor MMF creates the following induction harmonic wave

$$b_{\mu_a} = B_{\mu_a}\sin(1+(\mu_a-\nu_a)(1-s)\omega_0 t) = B_{\mu_a}\sin(\omega_0 t + eZ_2/p\cdot(1-s))\omega_0 t \tag{19}$$

where s slip of rotor of the main machine

The formula was derived based on equations reduced to the stator, i.e. it was written in a coordinate system fixed to the stator. The formula takes into account that the number of poles of the small synchronous machine has increased by a factor of eZ_2/p compared to the number of poles of the basic machine.

The stator current is written in the form

$$i_s e^{j\omega_0 t}$$

where ω_0 is the angular velocity of the mains voltage according to the fact that harmonics ν_b of the stator MMF induce a mains frequency voltage. The harmonic field of the stator ν_b itself rotates with a *mechanical* speed ω_0/ν_b, the harmonic field of the rotor μ_a rotates with the *same speed* compared to the rotor, ω_0/μ_a; therefore, if the rotor is in standstill (ω=0), the stator and the rotor fields are able to generate a constant torque. The rotor current is written in the form

$$i_r e^{j(\omega_0 t + eZ_2/p\cdot\omega_0 t)}$$

The description is further simplified by the substitution $\alpha = eZ_2/p\cdot(1-s)$.

Flux equations

$$\Psi_s = i_s e^{j\omega_0 t}L_s + i_r e^{j\omega_0 t}e^{j\alpha\omega_0 t}L_m \tag{20}$$

$$\Psi_r = i_s e^{j\omega_0 t}L_m + i_r e^{j\omega_0 t}e^{j\alpha\omega_0 t}L_r$$

The induced voltages are written in the *transformed* form

$$u\cdot e^{-j\omega_0 t}$$

for the sake of a simple equation:

$$u_s e^{-j\omega_0 t} = \frac{d\Psi_s}{dt}e^{-j\omega_0 t} = i_s j\omega_0 L_s + i_r(1+\alpha)j\omega_0 e^{j\alpha\omega_0 t}L_m \tag{21}$$

$$u_r e^{-j\omega_0 t} = \left(\frac{d\Psi_r}{dt} - j\omega\Psi_r\right)e^{-j\omega_0 t} =$$
$$= i_s j\omega_0 L_m + i_r(1+\alpha)\omega_0 j e^{j\alpha\omega_0 t}L_r - j\alpha\omega_0 i_s L_m - j\alpha\omega_0 i_r e^{j\alpha\omega_0 t}L_r =$$
$$= i_s(1-\alpha)j\omega_0 L_m + i_r j\omega_0 e^{j\alpha\omega_0 t}L_r$$

where the rotation speed ω of the rotor, due to the increase in the number of poles, is equivalent to the expression $\omega=\alpha\cdot\omega_0$.

Since the arrangement operates under forced current, the voltage is the output, resulting value. It can be seen that both the stator and the rotor have a rotational voltage of significant magnitude and frequency $\alpha\cdot f_{net}$. *This αtimes frequency is the reason why this whole phenomenon does not appear in the equivalent circuit even expanded with harmonic reactance.*

Near rated speed: $1+\alpha \approx 1-\alpha \approx e\cdot Z_2/p = e\cdot 2mq_2'$
The voltage: $u \approx i\cdot e\cdot 2mq_2'\cdot\omega_0\cdot L_m/\nu_b^2$

The torque is the vector product of the stator flux and the stator current (the operator *x* is intended to indicate this)

$$m = \Psi_s x i_s = i_s e^{j\omega_0 t}L_s x i_s e^{j\omega_0 t} + i_r e^{j\omega_0 t}e^{j\alpha\omega_0 t}x i_s e^{j\omega_0 t}L_m = \ldots =$$
$$= i_s i_r e^{j\alpha\omega_0 t}L_m = i_s i_r L_m\sin(\alpha\omega_0 t) \tag{22}$$

This torque is created in standstill if sign of ν_b and μ_a – that means rotation directions of stator and rotor - are identical in space: $\nu_b>0$, $\mu_a>0$ or $\nu_b<0$, $\mu_a<0$.

If the torque is created in rotation there are two possibilities.

If μ_a is negative (because e<0), necessarily $\nu_b>0$, the torque is created in the motoric range. Space rotation of rotor current shall be written as

$$i_r e^{-(j\omega_0 t + \alpha\omega_0 t)}$$

and the torque

$$m = i_s i_r L_m\sin((2-\alpha))\omega_0 t \tag{23a}$$

If μ_a is positive (because e>0), necessarily $\nu_b<0$, the torque is created in the brake range. Space rotation of stator current shall be written as

$$i_s e^{-j\omega_0 t}$$

and the torque

$$m = i_s i_r L_m\sin((2+\alpha))\omega_0 t \tag{23b}$$

The pulsating torque with constant peak means a linearly increasing peak power during the run-up, which is maintained by the also linearly increasing rotational induced voltage.

The voltage and torque oscillation frequencies are usually in the range of 600 Hz to 2000 Hz.

If the signs of μ_a and ν_b are different, the factor ξ_a/ξ_b must not be omitted.

Frequencies in (23), (23a) and (23b) correspond to the values summarized in the small table on p. 419 of [16], the latter shows the frequencies of the radial magnetic forces. This correspondence shows the *inherent* connection between the two phenomena. Since both derivations start from the *same* formula [19] (309), the results cannot be different.

If we calculate the synchronous parasitic torques created by the fundamental harmonic stator excitation, the rotor current is with a good approximation $i_r=-i_s\cos\varphi$, where φ is the angle between the rated voltage and rated current of the machine; otherwise $i_r=-i_s$.

To be substituted:
$I_s \approx 5$ p.u. starting current in the speed range of $s=1-s_{break}$
this means that in case of a possibly unfavorable rotor slot number, the high parasitic torque will oscillate with increasing frequency, but *of the same magnitude* (and the noise component resulting from the radial force occurring along with it) practically exists until the breakdown torque is reached
$I_s=1$ p.u. rated current on s_{rated} rated slip
$L_m=1/\omega_0\cdot X_m/\nu_b^2$ where $X_m \approx 3$ p.u.
Magnitude of induced voltage as example for
$e\cdot 2mq_2'=12$ and $\nu_b=13$ critical harmonic
resulting in very high synchronous parasitic torque

the range $s=1-s_{break}$ $u<5\cdot12\cdot3/13^2=1.06$ p.u.
on s_{rated} rated slip $u\approx1\cdot12\cdot3/13^2=0.21$ p.u.

8. Suitable Rotor Slot Numbers for Low-Noise 3-Phase and 5-Phase Machines

A radial force wave is directly related to the machine's magnetic energy through the proportionality factor C (3a), (3b). The magnitude of the same radial force wave has a natural relationship with its emission as noise power. Therefore, the expression of the proportionality factor C in dB does have a relation to the power of the noise excited by the respective force wave and with its human perception.

A part of Table 2. and Table 6. is repeated and put together below in Table 7. with C factors, therefore, re-calculated in dB; the 5-phase part is conveniently re-arranged. In both cases, the tables show only the largest components. The abbreviation n.a. in some cells means a very low level.

Of course, expressing the factor C (especially that of multiplied by an arbitrary number of 10^4) in dB has no any meaning in physic. Conclusions can and should be drawn not from the absolute values, but from the *difference* in the values expressed in dB on the same way as done generally with the noise power; also when used them for comparison of 3-phase and 5-phase machines.

Table 7: Proportionality Factors, Expressed in dB, for 3-Phase and 5-Phase Machines

Relative rotor slot number q_2'	Operation	C proportionality factor for radial magnetic force wave density f [N/m²] ·10000 in dB			Relative rotor slot number q_2'	C proportionality factor for radial magnetic force wave density f [N/m²] ·10000 in dB		
		$q_1=2$	$q_1=3$	$q_1=4$		$q_1=2$	$q_1=3$	$q_1=4$
q_1-2+**2/3**	motor	x	14,2	2,1				
	brake		11,7	7,4				
	standstill		3,7	9,2				
q_1-2+**5/6**	motor	x	-0,2	-3,4				
	brake		6,1	0,1				
	standstill		-3,1	n.a.				
q_1-1+**1/6**	motor	17,6	4,5	-0,4	q_1-2+**9/10**	x	-0,8	-6,6
	brake	4,9	5,3	-4,6	q_1-1+**1/10**	13,5	-1,5	-6,8
	standstill	4,4	-4,6	n.a.	q_1+**9/10**	4,5	-7,4	-11,0
q_1-1+**1/3**	motor	17,2	11,1	7,6	q_1-2+**4/5**	x	4,9	0,8
	brake	9,5	3,2	-0,3	q_1-1+**1/5**	11,1	4,6	0,9
	standstill	15,3	0,8	1,2	q_1+**4/5**	2,7	-0,8	-3,0
q_1+**2/3**	motor	4,3	-0,2	-2,8	q_1-2+**3/5**	x	10,1	2,0
	brake	9,5	6,1	3,9	q_1-1+**2/5**	5,9	-3,5	-6,8
	standstill	9,2	-5,2	-1,7	q_1+**3/5**	1,6	-0,2	-9,2
q_1+**5/6**	motor	-2,4	0,6	-8,0	q_1-2+**7/10**	x	-5,7	-4,2
	brake	9,0	-1,0	-4,4	q_1-1+**3/10**	2,0	0,8	-6,0
	standstill	-3,4	n.a.	n.a.	q_1+**7/10**	-4,2	-3,6	-9,2
					q_1±**1/2**	7,0	-3,0	-10,0

Let us take an example of comparing rotor slot numbers $q_1=2$, $q_2'=q_1-1+1/3$ vs. $q_2'=q_1+5/6$ in 3-phase. The C factors expressed in dB are for the first case number 17.2 dB, 9.5 dB, 15.3 dB, respectively, those for the second one are -2.4 dB, 9.0 dB, -3.4 dB, respectively. The *difference* between the noise excitation powers of the three power waves highlighted here are 19.6 dB, 0.5 dB, 18.7 dB, respectively. From this, the expected noise level *difference* of the two rotor slots compared to each other can really be deduced. Therefore the latter will exert much less noise because this short analysis is clearly highlighted the difference between the largest three of the many radial magnetic force waves generating the largest noise pressure waves; although the rest of waves further the frequencies of them are not included in the analysis at the moment, the conclusion is still valid.

Based on the above, a proposal is made hereby to introduce a general *Noise Component Equivalence Measure*. General definition

$$NCEM = 10\lg 10^4 \frac{\xi_{1\nu a}\xi_{1\nu b}}{\nu_b \mu_a}\eta_{2a}^2 \frac{1}{\xi_1^2} \tag{24}$$

for the noise component created by the fundamental harmonic only, appr.

$$NCEM = 10\cdot\lg 10^4 \frac{\xi_{1\nu b}}{\nu_b \mu_a} \tag{24a}$$

Here ξ, ν, μ and η^2 are real physical quantities. The 10^4 factor has no any physical meaning; its role is only that the resulting numerical value results in an order of magnitude convenient for human perception. Table 7 shows that a large positive value of the metric indicates a dangerous noise component, therefore the relevant slot number is not recommended, while a small positive and mainly a negative value is harmless and therefore indicates a recommended slot number. By calculating the metric for each significant excitation force wave and then including it in a single table, a complete, comprehensive and transparent picture of rotor slot number under investigation is obtained, which is thus more suitable than ever any previous method for judging the respective slot number. Those slot numbers that show high NCEM numbers are not worth investigating further with more advanced methods. We note that the applied factor could have been chosen just as arbitrarily for 10^{12}, so that the metric falls within the range of usual noise levels; this would give, however, the false impression, that the formula actually gives the expected noise level itself, indeed, so we deliberately do not do this.

$Z_2>Z_1$ is always recommended rather than $Z_2<Z_1$ from noise point of view [14].

Here too, we note that the reason for the outstanding values occurring in some cells in the tables is that the corresponding ν_b happens to be a slot harmonic.

It is strikingly apparent that, in 3-phase in case of some popular slot numbers, low-noise operation cannot be expected at all due to the noise component caused by the high synchronous torque component during brake mode; it is a torque that is not perceived during normal start-up and therefore generally not taken notice of, no attention is paid to it.

8.1. Design process of low-noise induction machines, evaluation of rotor slot number

The method is presented for a three-phase, 4-pole machine with a rotor slot number $Z_2=28$, which is otherwise inexplicably popular acc. to opinion of the author.

The design process starts with Table 1, which shows the synchronous parasitic torques generated in the case of the investigated rotor slot number, in order to check whether the machine will be able to run up at all. The chosen $q_2'=2+1/3$ relative number of slots in 4 poles means just $Z_2=28$ rotor slot number. The table also provides some preliminary information by calculating the noise proportionally factor for the components created by the radial magnetic forces corresponding to the synchronous parasitic torques.

Table 8: Order Number and Frequency (at 50 Hz) of Radial Magnetic Force Waves Generated by Rotor Slot Numbers, in 3-phase, 4-pole Machines; just investigated Z_2=28 Rotor Slot Number Highlighted

				p=	2												
				r=	0			r=	1	r=	2	r=	3	r=	4		
				+	-			+	-	+	-	+	-	+	-		
q_1	Z_1	ν_b	ν'_b			μ_a	μ_a									μ_a	μ_a
2	24	-11	-22	20	24			21	25	22	26	23	27	24	28		-13
			freq.	600	600			625	625	650	650	675	675	700	700		
		-11	-22	20	24			19	23	18	22	17	21	16	20		
			freq.	600	600			575	575	550	550	525	525	500	500		
		13	26	28	24	-13		29	25	30	26	31	27	32	28		15
			freq.	600	600			625	625	650	650	675	675	700	700		
		13	26	28	24	-13		27	23	26	22	25	21	24	20		
			freq.	600	600			575	575	550	550	525	525	500	500		
3	36	-17	-34	32	36			33	37	34	38	35	39	36	40		
			freq.	900	900			925	925	950	950	975	975	1000	1000		
		-17	-34	32	36			31	35	30	34	29	33	28	32	15	
			freq.	900	900			875	875	850	850	825	825	800	800		
		19	38	40	36			41	37	42	38	43	39	44	40		
			freq.	900	900			925	925	950	950	975	975	1000	1000		
		19	38	40	36			39	35	38	34	37	33	36	32		
			freq.	900	900			875	875	850	850	825	825	800	800		
4	48	-23	-46	44	48			45	49	46	50	47	51	48	52		
			freq.	1200	1200			1225	1225	1250	1250	1275	1275	1300	1300		
		-23	-46	44	48			43	47	42	46	41	45	40	44		
			freq.	1200	1200			1175	1175	1150	1150	1125	1125	1100	1100		
		25	50	52	48			53	49	54	50	55	51	56	52	-27	
			freq.	1200	1200			1225	1225	1250	1250	1275	1275	1300	1300		
		25	50	52	48			51	47	50	46	49	45	48	44		
			freq.	1200	1200			1175	1175	1150	1150	1125	1125	1100	1100		
5	60	-29	-58	56	60	29		57	61	58	62	59	63	60	64		
			freq.	1500	1500			1525	1525	1550	1550	1575	1575	1600	1600		
		-29	-58	56	60	29		55	59	54	58	53	57	52	56		-27
			freq.	1500	1500			1475	1475	1450	1450	1425	1425	1400	1400		
		31	62	64	60			65	61	66	62	67	63	68	64	-33	33
			freq.	1500	1500			1525	1525	1550	1550	1575	1575	1600	1600		
		31	62	64	60	-31	31	63	59	62	58	61	57	60	56		29
			freq.	1500	1500			1475	1475	1450	1450	1425	1425	1400	1400		
7	84	-41	-82	80	84		-41	81	85	82	86	83	87	84	88	43	
			freq.	2100	2100			2125	2125	2150	2150	2175	2175	2200	2200		
		-41	-82	80	84		-41	79	83	78	82	77	81	76	80		
			freq.	2100	2100			2075	2075	2050	2050	2025	2025	2000	2000		
		43	86	88	84		43	89	85	90	86	91	87	92	88		
			freq.	2100	2100			2125	2125	2150	2150	2175	2175	2200	2200		
		43	86	88	84		43	87	83	86	82	85	81	84	80	-41	
			freq.	2100	2100			2075	2075	2050	2050	2025	2025	2000	2000		
9	108	-53	-106	104	108			105	109	106	110	107	111	108	112		-55
			freq.	2700	2700			2725	2725	2750	2750	2775	2775	2800	2800		
		-53	-106	104	108			103	107	102	106	101	105	100	104		
			freq.	2700	2700			2675	2675	2650	2650	2625	2625	2600	2600		
		55	110	112	108	-55		113	109	114	110	115	111	116	112		57
			freq.	2700	2700			2725	2725	2750	2750	2775	2775	2800	2800		
		55	110	112	108	-55	55	111	107	110	106	109	105	108	104		
			freq.	2700	2700			2675	2675	2650	2650	2625	2625	2600	2600		
10	120	-59	-118	116	120			117	121	118	122	119	123	120	124		
			freq.	3000	3000			3025	3025	3050	3050	3075	3075	3100	3100		
		-59	-118	116	120			115	119	114	118	113	117	112	116	57	
			freq.	3000	3000			2975	2975	2950	2950	2925	2925	2900	2900		
		61	122	124	120			125	121	126	122	127	123	128	124		
			freq.	3000	3000			3025	3025	3050	3050	3075	3075	3100	3100		
		61	122	124	120			123	119	122	118	121	117	120	116		
			freq.	3000	3000			2975	2975	2950	2950	2925	2925	2900	2900		
11	132	-65	-130	128	132			129	133	130	134	131	135	132	136		
			freq.	3300	3300			3325	3325	3350	3350	3375	3375	3400	3400		
		-65	-130	128	132			127	131	126	130	125	129	124	128		
			freq.	3300	3300			3275	3275	3250	3250	3225	3225	3200	3200		
		67	134	136	132			137	133	138	134	139	135	140	136	-69	
			freq.	3300	3300			3325	3325	3350	3350	3375	3375	3400	3400		
		67	134	136	132			135	131	134	130	133	129	132	128		
			freq.	3300	3300			3275	3275	3250	3250	3225	3225	3200	3200		
12	144	-71	-142	140	144	71		141	145	142	146	143	147	144	148		
			freq.	3600	3600			3625	3625	3650	3650	3675	3675	3700	3700		
		-71	-142	140	144	71		139	143	138	142	137	141	136	140		-69
			freq.	3600	3600			3575	3575	3550	3550	3525	3525	3500	3500		
		73	146	148	144			149	145	150	146	151	147	152	148		
			freq.	3600	3600			3625	3625	3650	3650	3675	3675	3700	3700		
		73	146	148	144			147	143	146	142	145	141	144	140		71
			freq.	3600	3600			3575	3575	3550	3550	3525	3525	3500	3500		
14	168	-83	-166	164	168		-83	165	169	166	170	167	171	168	172	85	
			freq.	4200	4200			4225	4225	4250	4250	4275	4275	4300	4300		
		-83	-166	164	168		-83	163	167	162	166	161	165	160	164		
			freq.	4200	4200			4175	4175	4150	4150	4125	4125	4100	4100		
		85	170	172	168		85	173	169	174	170	175	171	176	172		
			freq.	4200	4200			4225	4225	4250	4250	4275	4275	4300	4300		
		85	170	172	168		85	171	167	170	166	169	165	168	164	-83	
			freq.	4200	4200			4175	4175	4150	4150	4125	4125	4100	4100		

Then Table IV. of [16] follows being *essential* for any noise calculation. Such a table is required for each pole number.

The table for 4 pole is repeated now as Table 8., omitting the parts of no interest at the moment, highlighting the rotor slot number Z_2=28 and integer multiple thereof.

The cell of the table in which the investigated slot number or its integer multiple appears creates a radial force wave with the corresponding $\underline{r}$ order number, with the frequency under it (in the case of 50 Hz power supply). As it is well known, the harmonic ν_b appearing not only in the row of the actual q_1 but also in the row of integer multiples thereof are also slotharmonics.

The table was supplemented with the μ_a induction wave harmonic produced by the investigated rotor slot number. In the table - although it was compiled by itself, independently of other tables -, the correlation between the synchronous parasitic torques and the radial magnetic forces stands out, and even that the radial force wave originates from the torque occurring in the rotation or in standstill (the latter always in pairs). The investigated rotor slot number, in addition to the always-occurring r=0 force waves, happens to only create r=4 force waves. Skipping the details: "of 1/3q" and "of 2/3q" rotor slot numbers only create r=0, 4 order numbers, while "of 1/6q", "of 1/2q" and "of 5/6q" rotor slot numbers create r=0, 2, 4 order numbers in 4 poles.

There is no other task than to insert the rest of force waves found here, order r>0, into Table 1, obtaining Table 9.

Table 9: Harmonics of a Three-Phase Machine, the Created Synchronous Parasitic Torques and NCEM Values for the Case of 4 poles, q_2' = 2 1/3 (Z_2=28) Relative Rotor Slot Number

m=	3	ν_b				q_1			Synchronous Torque / Breakdown Torque			Noise Component Equvivalence Measure			Freq.
q_2	+1/3	$\nu_b=-\mu_a$	$\nu_b=\mu_a$	operation	order nr.	q_1=2	q_1=3	q_1=4	q_1=2	q_1=3	q_1=4	q_1=2	q_1=3	q_1=4	Hz
	e	ν_b	μ_a		r=										
2	-1	13		motoric	0	ξ_{slot}			-2,23	0,50	0,29	17,57	11,10	8,73	600
	-1	-11	13		4	ξ_{slot}						18,30	10,94	9,45	700
	1	13	15		4	ξ_{slot}						16,95	10,48	8,11	700
	1	-17	15		4		ξ_{slot}					10,06	15,76	7,91	800
	-2	25	-27		4	ξ_{slot}		ξ_{slot}				11,56	4,20	11,52	1300
	-2	-29	-27		4							5,19	3,55	4,19	1400
	2	-29		break	0				-0,27	0,18	0,21	4,88	3,24	3,88	1500
	2	31	29		4							4,59	3,84	2,44	1400
	3	-41	43		4							1,67	0,91	-0,49	2200
	-3	43	41		4							1,67	0,03	0,66	2000
	-3		-41	standstill	0				≈0	≈0	≈0	≈0	≈0	≈0	2100
	3		43												
	-4	-53	-55		4		ξ_{slot}					-0,52	5,18	-1,52	2800
	4	55	57		4		ξ_{slot}					-0,83	4,86	-2,99	2800
	-4	55		motoric	0		ξ_{slot}		-0,14	0,52	-0,09	-0,68	5,01	-2,83	2700
	4	-59	57		4	ξ_{slot}						4,43	-2,04	-4,41	2900
	-5	67	-69		4							-2,39	-3,15	-3,40	3400
	-5	-71	-69		4	ξ_{slot}	ξ_{slot}	ξ_{slot}				2,82	2,80	2,79	3500
	5	-71		break	0	ξ_{slot}	ξ_{slot}	ξ_{slot}	-0,41	-0,41	0,40	2,82	2,80	2,79	3600
	-5	73	71		4	ξ_{slot}	ξ_{slot}	ξ_{slot}				2,58	2,56	2,55	3500
	6	-83	85		4	ξ_{slot}						1,47	-5,89	-7,38	4300
	-6	85	-83		4	ξ_{slot}						1,26	-5,21	-7,58	4100
	-6		-83	standstill	0	ξ_{slot}			0,35	≈0	≈0	1,47	≈0	≈0	4200
	6		85						-0,34			1,26			
	-9		-125	standstill	0		ξ_{slot}		≈0	-0,23	≈0	≈0	-2,12	≈0	6300
	9		127							0,23			-2,25		
	-12		-167	standstill	0	ξ_{slot}		ξ_{slot}	-0,17	≈0	0,17	-4,60	≈0	-4,64	8400
	12		169						0,17		-0,17	-4,71		-4,75	

With Table 9., the *final table as the very point of present development* was achieved, which provides the radial excitation force spectra exactly corresponding to the noise spectra to be measured later in the Testroom. This table thus actually, *numerically*, provides a basis for the design engineer's decision regarding the rotor slot number investigated. For this very purpose, the values of the synchronous parasitic torque were let remain in the table.

Table 9. confirms our previous view that the average noise pressure level of the machine established in the factory test room is actually caused by only a few low order harmonics, therefore it is usually sufficient to work with a table smaller (less harmonics that means less row) than Table 9.

The table cannot be as general and comprehensive as Table 1. or Table 5., this can only apply to a single rotor slot number. However, the stator slot number can be arbitrary; all that is required is to insert the relevant winding factors in a suitable way. For other pole numbers, the additional tables published in [16] should be used or to create with little investment of time.

Finally, the resonance frequencies must be checked so that the resonance does not unduly amplify even one single component. The resonance frequencies depend on the machine size; it is an old knowledge (Jordan, Heller) that some slot numbers, therefore, may or may not be suitable depending on the size of the machine.

8.2. *Effect of rotor skewing on noise*

The effect of skew is discussed in detail in [18]. Using our results and new formulas, let's examine the effect of skew *numerically* in Table 10. The calculations were performed for skewing by both the rotor and the stator slot pitch. As expected, the effect of skewing is not uniform, it is different for each component, depending on whether the harmonic causing the respective noise component is a slot harmonic on the stator or not.

Table 10a: Harmonics of a Three-Phase Machine, the Created Synchronous Parasitic Torques and NCEM Values for the Case of 4 poles, $q_2' = 2\ 1/3$ (Z_2=28) Relative Rotor Slot Number with rotor slot skewing *by one rotor slot pitch*; stator winding: full pitch

m=	**3**	ν_b				q_1			Synchronous Torque / Breakdown Torque			**N**oise **C**omponent **E**quvivalence **M**easure			Freq.
q_2	**+1/3**	$\nu_b=-\mu_a$	$\nu_b=\mu_a$	operation	order nr.	$q_1=2$	$q_1=3$	$q_1=4$	$q_1=2$	$q_1=3$	$q_1=4$	$q_1=2$	$q_1=3$	$q_1=4$	Hz
	e	ν_b	μ_a		r=										
2	-1	13		motoric	0	ξ_{slot}			-0,17	0,04	0,02	6,39	-0,08	-2,45	600
	-1	-11	13		4	ξ_{slot}						7,12	-0,24	-1,72	700
	1	13	15		4	ξ_{slot}						5,15	-1,32	-3,69	700
	1	-17	15		4		ξ_{slot}					-1,73	3,96	-3,89	800
	-2	25	-27		4	ξ_{slot}		ξ_{slot}				-2,79	-10,15	-2,83	1300
	-2	-29	-27		4							-9,16	-10,80	-10,16	1400
	2	-29		break	0				-0,01	0,01	0,01	-9,78	-11,42	-10,78	1500
	2	31	29		4							-10,07	-10,82	-12,22	1400
	3	-41	43		4							-14,70	-15,46	-16,86	2200
	-3	43	41		4							-14,50	-16,14	-15,50	2000
	-3		-41	standstill	0				≈0	≈0	≈0	≈0	≈0	≈0	2100
	3		43												

Table 10b: Harmonics of a Three-Phase Machine, the Created Synchronous Parasitic Torques and NCEM Values for the Case of 4 poles, $q_2' = 2\ 1/3$ (Z_2=28) Relative Rotor Slot Number with rotor slot skewing *by one stator slot pitch*; stator winding: full pitch

m=	**3**	ν_b				q_1			Synchronous Torque / Breakdown Torque			**N**oise **C**omponent **E**quvivalence **M**easure			Freq.
q_2	**+1/3**	$\nu_b=-\mu_a$	$\nu_b=\mu_a$	operation	order nr.	$q_1=2$	$q_1=3$	$q_1=4$	$q_1=2$	$q_1=3$	$q_1=4$	$q_1=2$	$q_1=3$	$q_1=4$	Hz
	e	ν_b	μ_a		r=										
2	-1	13		motoric	0	ξ_{slot}			-0,17	0,17	0,17	6,38	6,38	6,38	600
	-1	-11	13		4	ξ_{slot}						7,11	6,22	7,11	700
	1	13	15		4	ξ_{slot}						9,50	3,29	4,83	700
	1	-17	15		4		ξ_{slot}					2,62	8,57	4,63	800
	-2	25	-27		4	ξ_{slot}		ξ_{slot}				1,56	-2,54	1,86	1300
	-2	-29	-27		4							-4,81	-3,18	-5,47	1400
	2	-29		break	0				-0,03	0,03	0,03	-4,07	-4,07	-4,07	1500
	2	31	29		4							-4,36	-3,47	-5,51	1400
	3	-41	43		4							-9,00	-8,11	-10,15	2200
	-3	43	41		4							-8,79	-9,68	-7,64	2000
	-3		-41	standstill	0				≈0	≈0	≈0	≈0	≈0	≈0	2100
	3		43												

The numerical equalization occurring in Table 10b that means residual synchronous torque after skewing is always the same being independent from q_1 is derived by formula in [18].

It is not to forget that the decreasing factor shall be calculated acc. to μ_a: $\Delta_{rotor} = \sin(\mu_a\beta)/(\mu_a\beta)$

Comparing the tables, it is worth noting that in this case skewing by one rotor slot pitch gives better results.

When calculating the effect of slot skew the modification of the leakage reactance shall also be considered [18]; the tables in section 6.2, however, does not contain this for the moment.

9. Summary

This paper is a synthetic, closing work; our results before and now are sufficient for providing a complete foundation for achieving the final goal, the correct rotor slot number. All 3+1 formulas derided by us before [13], [14], [18] have been used for the purpose.

The tables developed in [16] for three-phase machines were further developed by substitution of the data of a typical machine into our formulas. We then used all of these to select the rotor slot number for the increasingly common 5-phase motors as well. It is shown that inverter fed multiphase motor's rotor slot number must be chosen as if it were a mains-supplied machine.

We worked with a relative rotor slot number and with p.u. data of a typical motor, which enables a systematic and comprehensive approach to the question and the incorporation of the entire range of rotor slot numbers for any pole. We have specified the favorable, acceptable and prohibited relative rotor slot numbers for 3-phase for the most common relative stator slot numbers regarding synchronous parasitic torque. The method can be transferred to 5-phase and any higher number of phases. At this point, a detailed, comprehensive recommendation, actual design rules are given the engineer on a new basis for how to select the rotor slot number for the entire range. The entire investigation relies on the basic law formulated by us that it is the p/r proper fractional part of the relative rotor slot number which determines the behavior of the asynchronous machine.

Then we dealt with the calculation of the noise-exciting radial force waves, in order to check the number of rotor slots resulting in low-noise motors. Relying on our formula, we have defined and introduced what we call the Noise Component Equivalence Measure (NCEM). This made it possible to accurately calculate the comparative spectra of the complete, noise-exciting radial forces; this enables the design engineer to select the most favorable rotor slot number and to exclude the a priori unfavorable rotor slot numbers from further examination.

Using the NCEM value and our results in [18] the effect of the rotor slot skewing was accurately calculated. The effect of skewing is not uniform, but different for each noise component.

In this study, we continued to follow the approach of starting the investigation from the rotor slot number. The reason for this is that the harmonics of the stator MMF (at least when q_1=integer) are always the same; on the contrary, the MMF harmonic order numbers of the rotor *depend* on the rotor slot number. This manner of vision made it possible to recognize relationships and laws that cannot be found in the works so far. The stator slot number affects the processes only to the extent that the actual slot harmonic amplifies some individual torque or noise components and others not, but the basic characteristics are determined by the rotor slot number.

The above summarized results should indicate the direction of further research using more advanced methods and measurements in terms of which rotor slot numbers are worth investigating further and which ones will definitely not be suitable.

With present study the entire range of rotor slot numbers in terms of parasitic torques and radial magnetic forces of all squirrel cage induction motors as a whole is covered on a comprehensive way being not available so far.

The conclusion and contribution to the research consists in the fact that thanks to our new formulas it is possible for the first time to give the machine designer a completely comprehensive, numerical recommendation for the entire range of asynchronous machines, for any size, for each number of poles and for each number of stator slots in order to select the correct number of slots for the rotor. Also for the first time, it was possible to numerically map the radial, noise-generating force waves according to the spectrum measured in the test room, for not skewed and skewed machines.

The entire examination of us on the effects of space harmonics in squirrel cage induction motors has come now to an end.

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Conflicts of Interest

The author declares no conflicts of interest.

Appendix

MMF harmonics in squirrel cage induction machines

The basic formulas for the order number of MMF space harmonics:

$$\nu_a = 6g_1 + 1 \quad (25a)$$
$$\mu_a = e \cdot Z_2/p + \nu_a = e \cdot 2mq_2' + \nu_a \quad (25b)$$
$$\nu_b = 6g_2 + 1 \quad (26)$$

The rotor harmonics generated by the fundamental MMF harmonic $\nu_a = 1$

$$\mu_a = e \cdot Z_2/p + 1 = e \cdot 2mq_2' + 1 \quad (27)$$

The harmonics of the stator MMF called here ν_b are considered now for $q_1 = 2$, together with the response of the rotor

$\nu_b = -5 \qquad 0<\eta_{2,-5}^2<1$
$\nu_b = 7 \qquad 0<\eta_{2,7}^2< 1$
$\nu_b = -11 \qquad \eta_{2,-11}^2 \approx 0$
$\nu_b = 13 \qquad \eta_{2,13}^2 \approx 0$

The magnitude of these harmonic fields is attenuated due to response of the rotor by the Δ_ν rotor attenuation factor [19] (269) p. 154:

$\nu_b = -5 \qquad \xi_5 \qquad \xi_5 \cdot \Delta_5 = \xi_5 \cdot (1- \eta_{2,-5}^2)$
$\nu_b = 7 \qquad \xi_7 \qquad \xi_7 \cdot \Delta_7 = \xi_7 \cdot (1- \eta_{2,7}^2)$
$\nu_b = -11 \qquad \xi_{11} = \xi_1 \qquad \xi_{11} \cdot \Delta_{11} = \xi_{11} \cdot (1- \eta_{2,-11}^2)\ (= \xi_1)$
$\nu_b = 13 \qquad \xi_{13} = \xi_1 \qquad \xi_{13} \cdot \Delta_{13} = \xi_{13} \cdot (1- \eta_{2,13}^2)\ (= \xi_1)$

The *apparent* winding factor of the slot harmonics remains the same, as it generates only a small current in the rotor to attenuate it.

The stator harmonics according to (25a) and (26), which are both numerically and physically *identical*, are called ν_a or ν_b according to their roles. If we examine the creation and order number of the harmonics of the rotor, specifically the $\mu_a = f(\nu_a)$ function as (25b), we use the marking ν_a for the stator harmonics; if these are examined by themselves, in a role independent of the order number of the rotor harmonics as in (26), then the marking ν_b is applied to them, as described above. From this marking, it is stated that if among the harmonics of rotor μ_a created by ν_a there is one for which $\nu_b=\pm\mu_a$ and $\nu_b\neq\nu_a$, then these harmonics form a synchronous parasitic torque.

On the other hand, the same harmonics marked as ν_a also create additional rotor harmonics:

$$\begin{aligned} \nu_a &= -5 & \mu_a &= e \cdot Z_2/p + \nu_a = e \cdot 2mq_2' - 5 \\ \nu_a &= 7 & \mu_a &= e \cdot Z_2/p + \nu_a = e \cdot 2mq_2' + 7 \\ \nu_a &= -11 & \mu_a &= e \cdot Z_2/p + \nu_a = e \cdot 2mq_2' - 11 \\ \nu_a &= 13 & \mu_a &= e \cdot Z_2/p + \nu_a = e \cdot 2mq_2' + 13 \end{aligned} \quad (28)$$

By substituting e=0 into formulas (28), we obtain the rotor harmonics that produce asynchronous parasitic torque *in any case*; for these, $\nu_b=\nu_a$. In the author's opinion, instead of equality of harmonics [22], it should be said here that ν_b does not appear in this physical phenomenon, it has no role whatsoever. These currents flow in the small harmonic circuits and are therefore part of the equivalent circuit diagram ([15] Figure 2).

By substituting e=±1, ±2, ±3,. into the formulas (28), we obtain the rotor harmonics that *can* produce synchronous parasitic torque with a ν_b other than ν_a. Only here does ν_b (26) enter this phenomenon. The role, impact and behavior of these fields being out of usual equivalent circuit diagram are presented in Chapter 7.

If $\nu_b=\mu_a$, then the torques are always generated in pairs and in standstill; if $\nu_b = -\mu_a$, the torque is generated "alone" and in rotation.

These further harmonics and their action can be studied "clearly," they can be separated from the action of the harmonics of the fundamental stator current harmonic if the rotor rotates synchronously. That is, if the rotor is rotated by a synchronous motor powered by the same network with the same number of poles. In this case, no fundamental harmonic rotor current occurs, and only currents generated by the stator harmonics flow. The coincidence of these further μ_a harmonics in (28) with any ν_b harmonic in (26) produces further synchronous parasitic torques and radial magnetic force waves. Therefore, in the proposed testroom arrangement, only these will work; however, they are excited by the low magnetizing current instead of the high starting current. In any case, in case of increased supply voltage (using saturation) upto the rated current, the effect of the investigated phenomenon on the rated operation can be accurately modeled. In most cases, especially with chorded winding the effect of rotor harmonics generated by stator harmonics other than fundamental harmonic is insignificant. In the case of skew they are sure insignificant.

Contrary to the perception so far *it is the q_2' relative number of rotor slots (see Figure 1.) which determines* at the end *how many* of the infinite number of *harmonics* occurring in the machine acc.

to (28) and their reaction and repeated counter-reaction *should actually be considered* when calculating the asynchronous machine with a cage rotor; the smaller q_2', the less should be considered. The result is that significantly fewer harmonics than those usually applied in by researchers so far.

Conflict of Interest

The author declares no conflict of interest.

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