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Analytical Study on the Effect of Rotor Slot Skewing on the Parasitic Torques of the Squirrel Cage Induction Machine

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A R T I C L E   I N F O

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A B S T R A C T

Skewing of the rotor slot of a squirrel-cage induction machine has been commonly used since the beginning. However, little guidance is given regarding the principle of the working effect of the method, but even in such rare cases, the explanation does not cover the true physical reality. Consequently no formula exists for calculation of the effect. In this study, the principle of the working effect of the rotor's skewing is derived in accordance with the true physics of the phenomenon, for both the synchronous and the asynchronous parasitic torque. The calculation regarding synchronous parasitic torque is based on comparing the stepped MMF curve of the straight rotor slot and the trapezoidal MMF curve of the skewed slot. New formulas are provided for both type of parasitic torques. Practical cases are investigated in details. Knowing the theory and the formula of slot skewing, the skew can now be applied on a targeted manner and in some cases, can completely eliminate the dangerous torque and noise components. Consequently, a theorem is formulated on when to skew according to the stator and when acc. to the rotor slot pitch; another theorem was found regarding residual synchronous parasitic torque after skewing. The investigation is then extended to include differential leakage attenuation calculations. With this in mind, from the point of view of rotor slot skew, the topic has been reviewed in its entirety.

1. Introduction

The skewing as a tool against harmful effects of stator slot harmonics is generally used by designer engineers in the industry therefore the topic is discussed in every book dealing with electrical machines. However, all such investigations, which use an analytical method, are aimed only at showing and calculating the effect on the leakage reactance of the machine. This is stated to be independent of whether the stator slot or the rotor slot is skewed. As the stator slot skew really reduces the magnitude of the slot harmonic of the stator MMF to a negligible level, it is concluded that it would also be the case if the rotor slot is skewed for manufacturing reasons.

Even today, intensive research is being conducted on the subject, which proves that researchers are not satisfied with the objective stated above and the generally accepted simple explanation shown below.

The results of the research show that skewing the rotor slot is more or less effective, indeed, but at the same time, the results of the researchers are sometimes in contradiction with each other see e.g. [1] vs. [2]. Many researchers primarily try to present their new calculation method and only use slot skewing as an example, with which they try to prove the correctness of their more advanced methods, and the goal is often not to study the slot skewing itself [3]-[7]. They do not deal with the actual physical working principle of the rotor slot skew, and there is sometimes confusion regarding the effect on the synchronous and/or asynchronous parasitic torque. [8] is an experiment that tries to derive the question, correctly, from the rotor's MMF shape; however, the method itself is fundamentally different from ours.

The results from the skew itself and from the quadratic effect of the increase in the leakage reactance, reducing both the starting torque and parasitic torques in the same proportion, are not separated.

Analytic approach of the subject is missing.

The goal of this paper is therefore to apply an analytical method, in order to fill the gap and facilitate better understanding.

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In the following, first the true principle of working for the rotor slot skew will be presented: a clear distinction is made between the effect on the synchronous and on the asynchronous parasitic torque, the difference in physics is clearly explained, then new formulas are derived, and calculations are performed. Recommendations are formulated as well.

Then the investigations are extended to the rest of the area of the effect of slot skew, that is, on attenuation of differential leakage. With this, the topic in its entirety is covered.

This paper strongly relies on our previous works [9]-[11]. The entire research is based on [12] as the very basic work of the electrical machine science history, some figures and calculations are copied from [13] and [14].

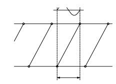
As a model, such a machine was assumed for which the basic formulas for determining the usual resulting space harmonics are valid [15]: infinite relative permeability, two-dimensional fields without considering boundary and end effects, the machine consist of two smooth coaxial cylinders made of magnetic material, the cylinders are separated by the air gap, the conductors of infinitely small cross-section are located in the air gap. Harmonics of other origin are not involved. Since the derivations as well as the conclusions drawn from the model are based on the fundamental laws of electrical engineering, they do not require validation. The essence of slot skew cannot be understood from FEM calculations, so there is no point in performing FEM calculations before analytical study. Still, as for validation the principle by measurements made by others see Section 9.

2. Symbols

R_1	stator ohmic resistance
$X_{s1} - X_{1\sigma}$	stator leakage without differential leakage reactance
X_m	magnetizing reactance
X_{s2}'	rotor leakage reactance
$X_{\sigma 2}'$	rotor differential leakage reactance
R_2'	rotor resistance reduced to stator
$X_{m\nu}=X_m \cdot 1/\nu^2 \cdot \xi\nu^2/\xi_1^2$	magnetizing reactance of harmonic ν
$X_{\sigma 2\nu}$	rotor differential leakage reactance of harmonic circuit ν
$R_{2\nu}=R_2 \cdot \xi_\nu^2/\xi_1^2$	rotor resistance of harmonic circuit ν
ξ_1, ξ_ν	winding factor of fundamental wave, harmonic wave
$\eta_{2\nu}^2$	Jordan's coupling factor
Δ	attenuation factor, decreasing factor
s, s_ν	slip of rotor to fundamental harmonic of stator; to harmonic ν of stator
ν, μ	designation of stator space harmonics and rotor space harmonics
ε	small positive number $\varepsilon \ll 1$.
a, b	designation of harmonics in interaction
g_1, g_2	different integers
e	integer
p	number of pole pairs
τ_p	pole pitch
θ, θ_m	MMF of one pole
Z_1 , Z_2	stator / rotor slot number
m, m_1, m_2	number of phases, stator, rotor
q_1, q_2'	relative slot numbers: stator/rotor slot number per pole per phase.

3. Critique of Generally Accepted Explanation of the Principle of Effect of Rotor Slot Skew

The effect of operation is usually explained by a figure like Figure 1. The figure is accompanied by the explanation that it is advisable to skew the rotor slots by one stator slot pitch. In this case, the inducing effect of the so-called stator slot harmonics on the rotor bars will be almost completely ineffective.



$S = \tau_{h1}$

Figure 1: Sketch for explanation of principle of working of rotor bar skew, s – skew, τ_{h1} – stator slot pitch

Figure 1. shows an arrangement in which the bars of the rotor are skewed by just one stator slot pitch; since the $2mq_1$ (otherwise not occurring in reality) harmonic MMF of the stator along the length of the rotor bar completes one complete period, no voltage is induced in it. In other words, the voltage induced in the rotor bar and its phase position, while moving along the length of the bar, moves along a circle, which returns to the initial position upon reaching the end of the bar. The wavelengths of the neighboring harmonics, called $2mq_1 \pm 1$ slot harmonics, which occur in reality, are only slightly different from the previous one, so they only induce a negligible voltage, that is, their effect on the operation of the machine becomes negligible in this way. Instead of the relationship $\xi_{slot} = \xi_1$ so far, the *apparent* relationship $\xi_{slot} \ll \xi_1$ is created, the skew "disappears" the stator slot harmonic.

This explanation is incomplete and therefore misleading. Incomplete in the sense that it is (partially) true regarding asynchronous parasitic torque but it is completely false regarding synchronous parasitic torque. Still the method is generally applied (almost) exclusively against synchronous parasitic torques on an intuitive basis without really considering its true working principle.

Below, it is proven that *explanations and conclusions do not and cannot cover true and complete physical reality.*

The critique is based on Figure 2 which is discussed in every book dealing with asynchronous machines. Therefore it will be discussed here in detail.

The equivalent diagram indicates that an asynchronous machine consists of a series of small asynchronous machines in shaft connection [15]. Each circuit belongs to a ν harmonic of the stator MMF.

The stator harmonics in the diagram, on its "primary" side, appear with a known winding factor ξ_ν. The rotor affects this only to the extent that it *attenuates* some of them and not others. In accordance with practice, our diagram is drawn with the assumption that the stator and rotor slot numbers are not too far apart. In this case, harmonics of lower order than the first slot harmonic are attenuated, those higher than that are not attenuated [9].

It is a basic law that if a ν^{th} harmonic does not generate current in the cage, it means not more than it does not form an asynchronous parasitic torque with it. In this case, however, that ν^{th} not-attenuated harmonic remains in its *original size* and is still suitable for the formation of a synchronous parasitic torque because no current in the cage by a ν^{th} harmonic means no attenuation of that harmonic.

All this contradict the generally accepted explanation.

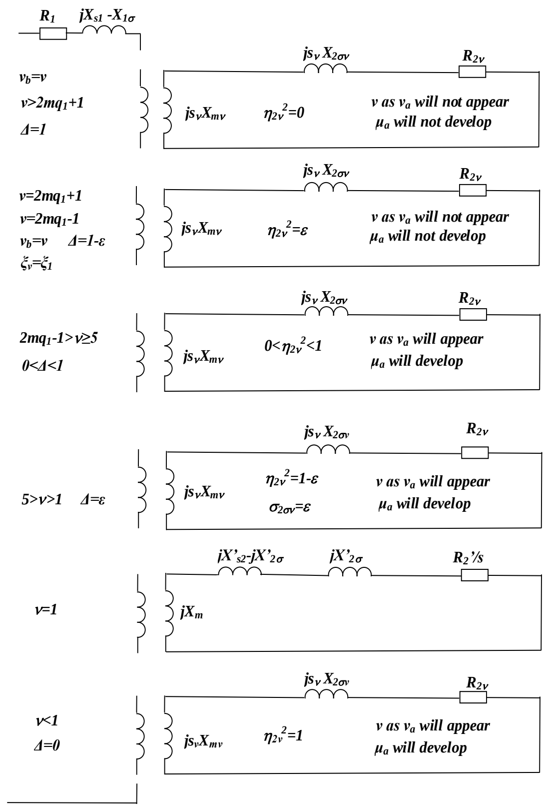


Figure 2: Equivalent circuit including the circuits for the higher harmonics [9]

In Figure 2., only the rotor differential leakage is included as a leakage in the harmonic equivalent circuits [12], [15]. Therefore, the value of the rotor differential leakage will determine how high current will flow in a small harmonic circuit. The differential leakage reactance is [12] (268)

$$X_{2\sigma\nu} = X_{m\nu} \cdot \sigma_{2\sigma\nu} \tag{1}$$

where

$$\sigma_{2\sigma\nu} = \frac{1}{\eta_{2\nu}^2} - 1$$

is the differential leakage factor of the rotor. The denominator in it is the so-called Jordan's coupling factor. Its definition ([12] 268b):

$$\eta_{2\nu_a} = \frac{\sin(\nu \frac{p\pi}{Z_2})}{\nu \frac{p\pi}{Z_2}} \tag{2}$$

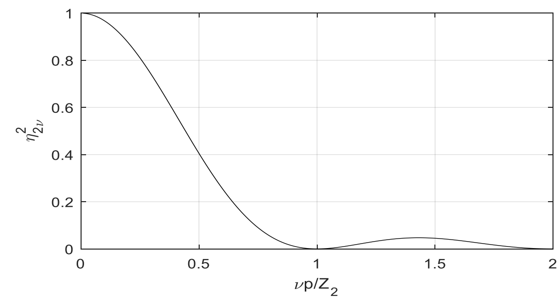


Figure 3: Plotting the value of $\eta_{2\nu}^2$ as a function of $\nu p/Z_2$ ([12] p.154. Figure 107.)

Equation (2) is plotted on Figure 3. It shows how much the rotor responds to a stator harmonic. A zero or very low value indicates that the rotor does not respond to that harmonic. Then the differential leakage factor will be very high or even infinite ([15] Figure 17. p. 44).

For a better illustration, this figure has been modified by us in [9], while the range $\nu p/Z_2>1$ was replaced by zero.

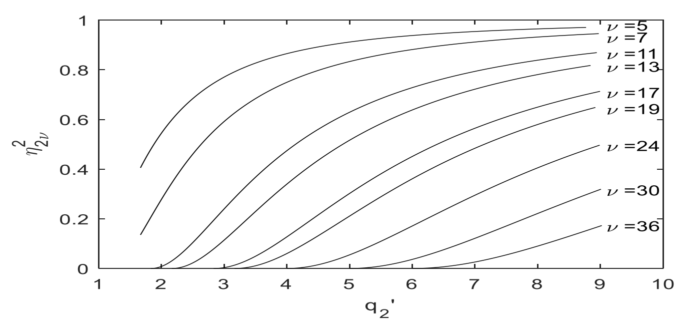


Figure 4: Representation of the value of $\eta_{2\nu}^2$ as a function of q_2' with ν as a parameter. Higher odd harmonics are replaced mathematically by adjacent (actually with 3-phase not existing) even harmonics only for better transparency

The application of the figure will be demonstrated on an example: if e.g. q_1=3, then the stator harmonics belonging to it are ν=17 and ν=19.

If then $q_2' \leq 3$, the rotor does not respond to these harmonics (and the harmonics higher than these) *no matter skewed or not skewed.*

If, on the other hand, $q_2'>3$, since $\eta_{2\nu}^2$ is no longer a very small value therefore $\sigma_{2\sigma\nu}$ is no longer an (infinitely) large value the rotor then also responds to the stator slot harmonics and thus a significant asynchronous parasitic torque is generated. This is the point when the slot skewing shall be introduced by the designer, so that the slot harmonic can only induce negligible voltage in the bar.

However, this phenomenon has nothing to do with the generation of synchronous parasitic torque, as it has a different physical basis.

In fact, the rotor skewing was introduced first – in accordance with the true physics – just for suppressing the asynchronous parasitic torques ([12] p. 181, p. 187.) in the case of $Z_2>Z_1$; the explanation added was clearly directed for that purpose: the

$Z_2<1.25 \cdot Z_1$ law was recommended just to prevent high asynchronous parasitic torque with straight bar otherwise slot skewing shall be applied. The solution was later applied (almost exclusively) against synchronous parasitic torques as well on an intuitive basis.

All books on the subject, including [12], explain the phenomenon based on a representation like Figure 1. It is clear that each such figure represents an arrangement where $Z_2<Z_1$. In this case, however, *no asynchronous parasitic torque is created therefore no need to suppress any such torque*. The representation is very close to the situation in which $\tau_{h1}=\tau_{h2}$ (i.e. $Z_2=Z_1$), when the rotor *by definition* does not respond to the slot harmonic of the stator *no matter skewed or not skewed*. *This manner of representation* drew the attention of the author to the fact that the generally accepted explanation *cannot be correct*.

Now the true and complete theory of the whole phenomenon will be presented in a logical sequence of topics for evidencing and better understanding. First, the physics of stator skew and then that of synchronous torque itself will be discussed, dispelling misconceptions; these lay the foundation for the derivation of the formulas. Then we turn to the rotor skew, studying practical cases as well. These help to find and formulate new theorems. Finally accompanying phenomena of the topic will be examined.

4. Skewing of Stator Slots

The slot skew is – acc. to traditional derivation - usually modeled as if the number of slots on an arc equal to the pitch of a slot was infinite. Figure 5. ([13] Figure 113.) was originally developed for the winding distribution factor.

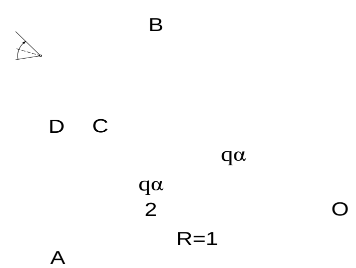


Figure 5: Calculation of the distribution winding factor for an infinite number of stator slots ([13] Figure 113.)

An infinite number of slots entails that the phase position of the voltage induced by the rotating main field in the winding-side sections that are infinitely close to each other moves along a circular arc. The $q\alpha$ range for the m=3 phase is $\pi/3$.

Thus, for a skew belonging to one slot pitch: $\alpha/2 = \pi/3 \cdot 1/2q_1 = \pi/6q_1$.

The skew factor acc. to this model (chord per arc)

$$\xi_{skew} = \frac{\sin(\alpha/2)}{\alpha/2} = \frac{\sin(\pi/6q_1)}{\pi/6q_1} \tag{3}$$

The skew factor for the fundamental harmonic is approximately 0.995, that is, the reduction of the main field with skewing by one stator slot pitch is not detectable small.

However, let us substitute the value of the first slot harmonic:

$$\xi_{skew\,\nu slot} = \frac{\sin(\nu_{slot} \cdot \pi/6q_1)}{\nu_{slot} \cdot \pi/6q_1} = \frac{\sin((2mq_1+1)\frac{\pi}{6q_1})}{(2mq_1+1)\frac{\pi}{6q_1}} = \ldots = \tag{4}$$

$$= \frac{\sin(\pi/6q_1)}{\pi(1+1/6q_1)} \approx \mp \frac{\sin(\pi/6q_1)}{\pi}$$

The numerator of the fraction is the same as that of the fundamental harmonic; however, its denominator is ~$6q_1$ times higher that means more than one order of magnitude larger. The value of $\xi_{skew\ \nu slot}$ for $q_1=2$ is approximately 7%, skewing the stator slot is really effective, it "disappears" the slot harmonic of the stator. The skewing of the pole of the synchronous machine can be modeled in the same manner.

A very vivid picture is obtained if the same phenomenon is approached differently.

Let us investigate the MMF curve of one phase of a 3-phase stator winding first for $q_1=1$ acc. to Figure 6. The curve non-skewed is represented by the solid line, that trapezoidal MMF curve created as a result of the stator slot skewing is represented by the dashed line.

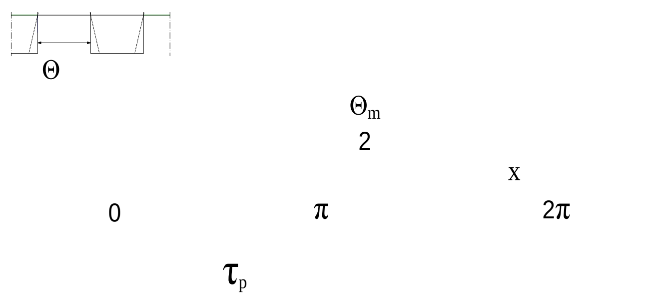


Figure 6: MMF curve of one phase of a three-phase winding, $q_1=1$.

Let us consider the harmonic content of both arrangements.

Fourier series of MMF created by two coil sides lying in non-skewed slots in a pole pitch distance as a function of location along the circumference see Figure 7. (solid line in Figure 6.)

$$\Theta(x) = \frac{4}{\pi}\frac{\Theta_m}{2}(\sin x + \frac{1}{3}\sin 3x + \ldots + \frac{1}{\nu_{slot}}\sin \nu_{slot} x + \ldots) \tag{5}$$

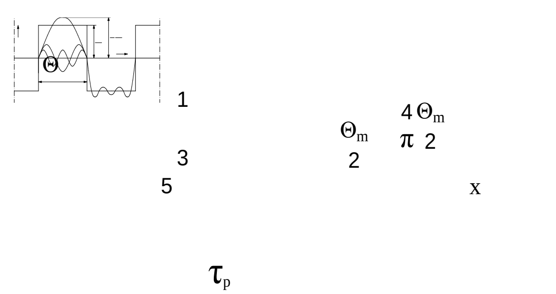


Figure 7: Harmonics of square MMF curve [13] Figure 151.

Let us consider Figure 8. It is originally elaborated for infinite number of stator slots [13]. It is not difficult to notice that the slot skewing by one stator slot pitch and the infinite number of stator slots result in the identical MMF curve.

The Fourier series of MMF created by two coil sides lying in skewed slots in a pole pitch distance as a function of location along the circumference (dashed line in Figure 6.) ([13] (99)

$$\Theta(x)=\frac{4}{\pi}\frac{\Theta_m}{2\beta}\left(\sin x\sin\beta+\frac{1}{3^3}\sin 3x\sin 3\beta+...+\frac{1}{\nu_{slot}^2}\sin\nu_{slot}x\sin\nu_{slot}\beta+...\right) \qquad (6)$$

For one phase of a three-phase winding, $\beta=\pi/3\cdot 1/2=\pi/6$, generally $\beta=\pi/2m_1$ see (21) later.

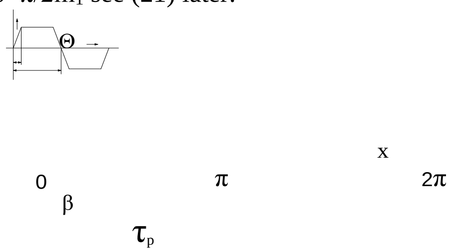


Figure 8: Trapezoidal MMF curve of infinite slot number (q=∞) ([13] Figure 152.)

Proportioning the harmonics formed according to (6) and (5), all harmonics including the fundamental harmonic decrease due to skewing by the factor Δ_{stator} as follows:

$$\Delta_{stator}=1/\beta\cdot 1/\nu\cdot \sin(\nu\beta) = \sin(\nu\beta)/(\nu\beta) \qquad (7)$$

Substituting $\beta=\pi/6$ for three-phase the formula for stator slot skewing by exactly one stator slot pitch is:

$$\Delta_{stator}=\frac{\sin\nu\beta}{\nu\beta}=\frac{\sin(\nu\cdot\frac{\pi}{6})}{\nu\frac{\pi}{6}}=\frac{1}{\nu}\frac{3}{\pi} \qquad (8)$$

for $\nu=1, 5, 7$, etc.

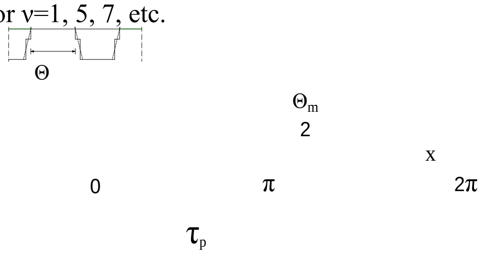


Figure 9: MMF curve of one phase of a three-phase winding, $q_1=3$, full pitch winding

Now the practical cases $q_1>1$ are examined, with q_1 integer, with full pitch winding, see Figure 9.

As it can be seen, the skewed line of one complete phase is the same, it is *independent* of the relative slot number q_1, but the measure of skew created by one coil in two slots is only one-third ($1/q_1$ times) of the previous one.

The formula of Fourier analysis of the solid line of Figure 9. is:

$$\Theta(x)=\frac{4}{\pi}\frac{\Theta_m}{2}\left(\xi_1\sin x+\frac{1}{3}\xi_3\sin 3x+\frac{1}{5}\xi_5\sin 5x+...+\frac{1}{\nu_{slot}}\xi_{\nu_{slot}}\sin\nu_{slot}x+...\right) \qquad (9)$$

where ξ_ν contains only the distribution factor. The ratio according to (7) has changed, since now the ratio of (6) and (9) shall be formed. The *basic* formula for stator MMF harmonic reduction factor due to slot skewing by exactly one stator slot pitch is:

$$\Delta_{stator}=\frac{1}{\xi_\nu}\frac{\sin\nu\beta}{\nu\beta}=\frac{1}{\xi_\nu}\frac{\sin\nu\cdot\pi/6}{\nu\cdot\pi/6}=\frac{1}{\xi_\nu}\frac{1}{\nu}\frac{3}{\pi} \qquad (10)$$

again for $\nu=1, 5, 7, ...$

Substituting for the ratio of the slot harmonics of the skewed and non-skewed slots:

$$\Delta_{stator}=\frac{1}{\xi_{slot}}\frac{\sin\nu_{slot}\beta}{\nu_{slot}\beta}=\frac{1}{\xi_1}\frac{\sin((2mq_1+1)\frac{\pi}{6})}{(2mq_1+1)\frac{\pi}{6}}=\frac{1}{\xi_1}\frac{\sin((2mq_1+1)\frac{\pi}{6})\frac{1}{q_1}}{(2mq_1+1)\frac{\pi}{6}\frac{1}{q_1}} \qquad (11)$$

Contrary to expectation, (3) and (11) are not identical, not only because of $\xi_{\nu slot}=\xi_1$ appeared in the formula. The reason for this can be found in Figure 9. The effect of skewing of each slot adds up, the phenomenon works *cumulatively*. This fact is not taken into account by the traditional approach; although the difference is small, theoretically it is not right to apply (3) for the stator skew.

Table 1 summarizes the Δ_{stator} MMF harmonic reduction factors calculated acc. to (10).

Table 1: Reduction Factor of Stator Harmonics by Skewing the Stator by one Stator Slot Pitch, for $q_1=1$, $q_1=2$ and $q_1=3$, full pitch winding

harmonic	ξ_ν	ξ_ν	reduction factor Δ		
ν	$q_1=2$	$q_1=3$	$q_1=1$	$q_1=2$	$q_1=3$
5	0,259	0,217	0,191	0,738	0,878
7	-0,259	-0,178	-0,136	0,527	0,769
11	-0,966	-0,178	-0,087	0,090	0,489
13	-0,966	0,217	0,073	-0,076	0,338
17	-0,259	0,960	0,056	-0,217	0,059
19	0,259	0,960	-0,050	-0,194	-0,052
23	0,966	0,217	-0,042	-0,043	-0,191
25	0,966	-0,178	0,038	0,040	-0,215

It is clear from the table that the respective slot harmonics are effectively reduced, but the non-slotharmonics are reduced only slightly/moderately. This means that slot skewing of the stator removes only the synchronous parasitic torques caused by slot harmonics; the torques produced by non-slot harmonics will be only moderately reduced. The non-slot harmonics are already small due to $\xi_{\nu\text{-non-slot}} \ll \xi_1$; those are only slightly reduced further by skewing.

Next would be the examination of chorded windings. These do not create an MMF curve according to Figure 9, but only a similar one, since the height of the "steps" will no longer be identical. Therefore (6) will not be valid; a closed formula cannot be provided. Because several versions of chording are possible, this would result in very extensive and not easily comprehensible tables, so we dispense with this examination for the moment.

If it is not skewed with exactly one stator slot pitch, then the MMF consists of stepped and trapezoidal sections, therefore a closed formula cannot be provided either. Since this way of skew does not happen in practice, we do not deal with it.

5. Synchronous Parasitic Torque

The synchronous parasitic torque is calculated by the circumferential integral of the product of the peripheral current layer $a_{\nu b}$ created by the stator current harmonic order ν_b and the induction $b_{\mu a}$ created by the rotor current harmonic order μ_a [12].

$$M \approx \int_0^{2p\tau} a_{\nu_b} b_{\mu_a} dx \qquad (12)$$

The symbol ≈ means here not more than the synchronous torque M is proportional to the integral. The integral differs from zero only, that means, a synchronous parasitic torque is generated only if $\nu_b = \pm\mu_a$. Regarding definitions of harmonics, reference is made to the Appendix.

The induction wave $b_{\mu a}$ is generated by the harmonic of the fundamental harmonic current MMF of the *rotor*. This harmonic current is *not part of the equivalent circuit diagram* because it induces fields with frequencies other than that of the net (much higher than that of the net) ([12] p. 133); therefore, it does not appear in the stator voltage equations. It depends solely on the fundamental harmonic current of the *rotor* because it is *its* harmonic; in particular, it does not depend on the stator current layer $a_{\nu b}$. If it were to depend, it would be an asynchronous phenomenon, and would thus be part of the equivalent circuit diagram.

The synchronous parasitic torque *cannot* therefore be explained on the basis of Figure 2. For this reason, the approach regarding synchronous parasitic torques that makes the effect of operation of the slot skew dependent whether or not the stator (slot) harmonic induces into the rotor cage loop is *incorrect*. The definition of the synchronous parasitic torque is clearly stated in [15]: “if there are harmonics of the same order in the spectra of the MMF harmonics of the stator and rotor, and this harmonic of the rotor is produced by another harmonic of the stator, $\nu_b \neq \nu_a$, (see Appendix) that is, they are *independent* (*supplement* by the author), then they form a synchronous torque. If it is produced by the same stator harmonic, that is, if $\nu_b = \nu_a$, that is, $b_{\mu a}$ *depends* on $a_{\nu b}$ (*supplement* by the author), then they form an asynchronous torque.” We therefore propose the introduction of a *modified* theorem, which has a stronger expressive power in terms of physical message.

The rotor harmonic induction $b_{\mu a}$ decreases as a result of rotor slot skew acc. to (7), the product formed by *unchanged* $a_{\nu b}$ decreases in the same ratio.

This is the true explanation of the principle of effect of rotor slot skew on the synchronous parasitic torque.

It is especially legitimate to speak here of the unchanged current layer wave $a_{\nu b}$ because, as we shall see in Chapter 9., when the rotor slot is skewed, an asynchronous parasitic torque will not occur, therefore no attenuation of the harmonics ν_b occurs.

Once again, the harmonic $a_{\nu b}$ of the stator current layer changes only if asynchronous parasitic torque is created *through* it; therefore, attenuation also occurs simultaneously. Therefore, the two phenomena, skewing and attenuation, must be treated separately.

Now the original equation [12] (310) p. 191., will be quoted in full which by definition is referred to the skewing of the rotor

$$m_{\mu_a \nu_b} = \frac{p\tau l_i}{\pi} \chi_{2\mu_a} \int_0^{2p\tau} a_{\nu_b} b_{\mu_a} dx_1 \qquad (13)$$

where

- $m_{\mu a \nu b}$ synchronous torque created by the interaction of harmonics $a_{\nu b}$ and $b_{\mu a}$
- 2p number of poles
- τ pole pitch; pτ/π is the radius of the rotor
- l_i ideal iron core length
- $\chi_{2\mu a}$ the slot skew factor relating to the harmonic number μ_a of the rotor, its definition for an arbitrary harmonic ([12] (206) p.123, (268) p. 154)

$$\chi_{2\nu} = \frac{\sin\left(\nu \frac{s}{\tau_{h2}} \frac{p\pi}{Z_2}\right)}{\nu \frac{s}{\tau_{h2}} \frac{p\pi}{Z_2}} \qquad (14)$$

x_1 is the perimeter location variable of the integration

After formal transformation of (13)

$$m_{\mu_a \nu_b} = \frac{p\tau l_i}{\pi} \chi_{2\mu_a} \int_0^{2p\tau} a_{\nu_b} b_{\mu_a} dx_1 = \frac{p\tau l_i}{\pi} \int_0^{2p\tau} a_{\mu_b} \left(\chi_{2\mu_a} b_{\mu_a}\right) dx_1 \qquad (15)$$

Converted in this way, the formula exactly corresponds to the true principle of working effect given above.

This means that [12] showed precisely the theory of effect of rotor skew on the synchronous parasitic torque even at that early time. However, Richter *did not add* any word of an explanation to the formula. It can be assumed that the formula, but especially its physical content, has *therefore* completely escaped the attention of researchers since then, so they have been forced to attempt to calculate the phenomenon using very sophisticated apparatus.

6. Skewing of Rotor Slots

Because q_2=1 per definition (see below) for a squirrel cage rotor the harmonics created by the straight bars are given by (5), and those created by the skewed bars are given by (6), the ratio of them is given by (7).

The rotor cage is a multiphase arrangement. The investigation is now conducted with the easiest arrangement, where $Z_2/2p$=integer. In this case, there is another bar of opposite phase at a distance of a pole pitch. With example of $q_1 = q_2' = 2$, the MMF of the rotor is as if it were an $m_2 = q_1 m_1 = 6$-phase, with identical $q_2 = 1$ winding system arrangement. In this section, q_2 represents the number of slots connected in series, producing MMF in phase. The slot numbers are expressed now:

$$Z_1 = 2pm_1q_1 \qquad (16a)$$

$$Z_2 = 2pm_2q_2 \qquad (16b)$$

where $q_2 \equiv 1$, which means that there are no cage bars connected in series, no MMF in phase are created. Further rearrangement by applying the usual definition of q_2'

$$Z_2=2pm_2=2pm_1q_2' \quad (17)$$

then $$m_2=m_1q_2' \quad (17a)$$

To date, the most important question is what β should be substituted if the Figure 8. is applied to rotor skew.

In Figures 10a–10d are therefore plotted the MMF curves of one phase of both the stator and rotor for q_1 = 1, 2, and 3 non-skewed and as if both side were skewed by one stator slot pitch at the same time; several rotors were drawn for each stator in the same figure to demonstrate all possibilities. The rotor slot number q_2'=2 1/3 is shown with both one stator slot pitch skew and one rotor slot pitch skew. In the rotor curves, the MMF curve of some adjacent bar pairs, that is, some rotor phases adjacent to the one under consideration, are also indicated. The scale of the MMF in the figures is not the same, allowing easier demonstration.

One stator slot pitch means

$$\tau_{h1}=\frac{2\pi}{2m_1q_1}=\frac{\pi}{m_1q_1} \quad \text{electrical degree} \quad (18)$$

β is half of this:

$$\beta=\frac{\pi}{2m_1q_1} \quad \text{electrical degree} \quad (19)$$

Consequently, regardless of the rotor slot number, if the rotor is skewed by one stator slot pitch β must be substituted according to (19). As shown in Figure 10, the same phenomenon was observed geometrically.

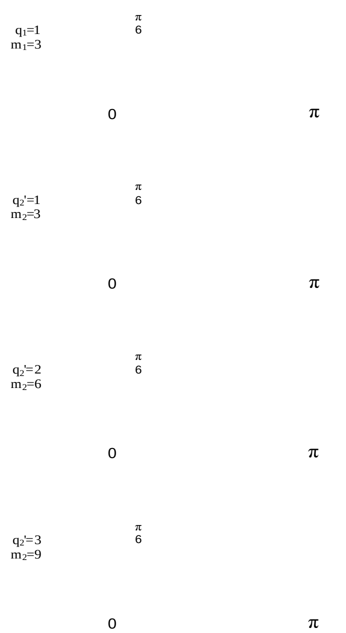


Figure 10 a: q_1=1 q_2'=1, 2, 3

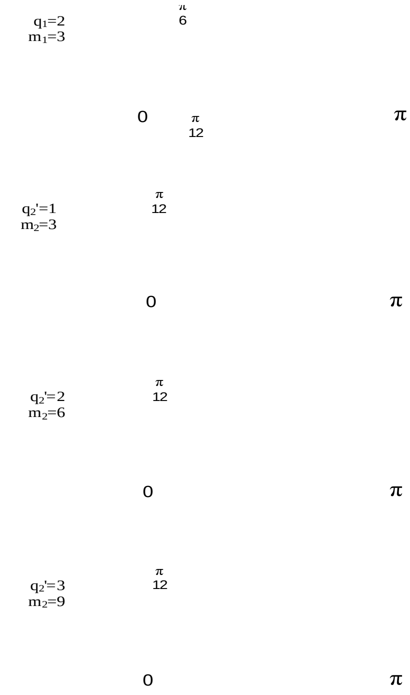


Figure 10 b: q_1=2 q_2'=1, 2, 3

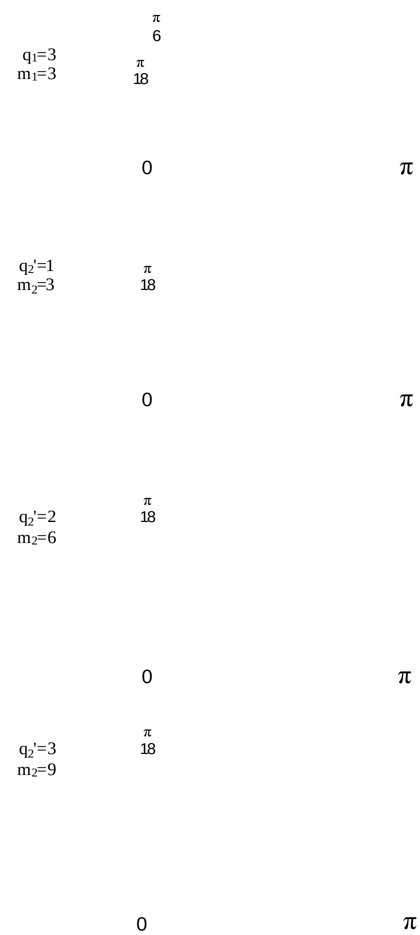


Figure 10 c: q_1=3 q_2'=1, 2, 3

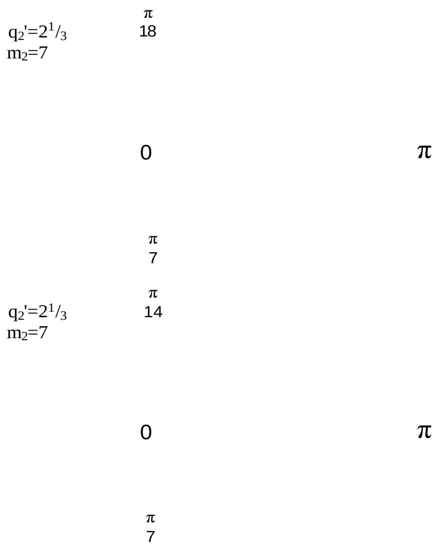


Figure 10 d : q_1=3 q_2'=2 1/3

Figure 10. MMF of one phase of skewed stator and skewed rotor put together in one figure

If the rotor is skewed by one rotor slot pitch

$$\beta = \frac{\tau_{h2}}{2} = \frac{\pi}{2m_1 q_2'} = \frac{\pi}{2m_2} \quad (20)$$

corresponding the remark to (6) before.

If the stator is skewed by one stator slot pitch

$$\beta = q_1\frac{\tau_{h1}}{2} = \ldots = \frac{\pi}{2m_1} \quad (21)$$

electric degrees. Physically, this occurs because q_1 on the stator also means that the phase of the excitation current in the slot belonging to q_1 slots is identical.

It can be seen that the same rotor slot number installed in a different stator produces a different MMF curve if the rotor is skewed by the actual stator slot pitch.

Table 2: Reduction Factor of Harmonics Produced by the Rotor by Skewing the Rotor by one Stator Slot Pitch

q_1	1	2	3
β	π/6	π/12	π/18
harmonic	reduction factor		
ν	Δ	Δ	Δ
5	0,191	0,738	0,878
7	-0,136	0,527	0,769
11	-0,087	0,090	0,489
13	0,073	-0,076	0,338
17	0,056	-0,217	0,059
19	-0,050	-0,194	-0,052
23	-0,042	-0,043	-0,191
25	0,038	0,040	-0,215

A Fourier analysis of the rotor MMF curves acc. to (24) is given in Table 2. The table provides a very important result. At the arrangement q_1=2, as shown in Figure 10b, β=π/12 acc. to (19). Analyzing the MMF curve of the rotor, only the rotor harmonics of μ= 11-13 and their integer multiples are significantly reduced; the rest of the harmonics are reduced only moderately. In Figure 10c, where acc. to (19) β=π/18, only the rotor harmonics μ=17-19 and their integer multiples are significantly reduced; the rest of the harmonics are only moderately reduced.

Formula (7) is therefore only for a slot harmonic

$$\Delta_{rotor} = \ldots\frac{1}{\beta}(\ldots + \frac{1}{\nu_{slot}}\sin(\nu_{slot}\beta) + \ldots) \quad (22)$$

Substituted

$$\Delta_{rotor} = \frac{\sin\nu_{slot}\beta}{\nu_{slot}\beta} = \frac{\sin((2mq_1 \pm 1)\cdot\frac{\pi}{6q_1})}{(2mq_1 \pm 1)\cdot\frac{\pi}{6q_1}} \approx \frac{\mp\sin(\frac{\pi}{6q_1})}{\pi} \quad (23)$$

being identical with (4) because the effect of rotor slot skewing is not cumulative.

In general, if ν is an arbitrary harmonic of the stator, the *basic* formula for the rotor skewing

$$\Delta_{rotor} = \frac{\sin\nu\,\beta}{\nu\,\beta} = \frac{\sin(\nu\cdot\frac{\pi}{6q_1})}{\nu\cdot\frac{\pi}{6q_1}} \quad (24)$$

The sin function in the numerator of the formula gives the phenomenon a specific periodicity in this way as it can be seen on Table 2.: the synchronous parasitic torques produced by non-stator slot harmonics will decrease less.

Regarding Table 1. and Table 2. it is surprising that the value of the Δ reduction factors are identical although the formulas are different. Therefore the phenomenon deserves further examination.

6.1 Remaining synchronous parasitic torque with slot skew by one stator slot pitch

Let's examine the resulting, *remaining* synchronous parasitic torque $M_{residual}$ for both stator skew and rotor skew.

The formula for the calculation of the synchronous torque, which was first derived by us [10] (4):

$$\frac{M_{synchronous}}{M_{breakdown}} = \frac{X_m}{X_s}\cdot 2\sum\frac{\xi_{1\nu_a}\xi_{1\nu_b}}{\mu_a}\eta^2_{2\nu_a}\frac{1}{\xi_1^2} \quad (25)$$

where

$$\eta_{2\nu} = \frac{\sin\nu\frac{p\pi}{Z_2}}{\nu\frac{p\pi}{Z_2}} = \frac{\sin\nu\frac{\pi}{2mq_2'}}{\nu\frac{\pi}{2mq_2'}}$$

M torque

X_m, X_s reactance, magnetizing, leakage

$\xi_{1\nu a}$, $\xi_{1\nu b}$, ξ_1 winding factor of harmonics, of fundamental harmonic

$\eta_{2\nu}$	Jordan's coupling factor
ν_a ,ν_b , μ_a	space harmonic order numbers stator, rotor
m	number of phases
Z_2	rotor slot number
2p	pole number
q_2'	relative rotor slot number

Now, a simplification and approximation $\xi_{1\nu a}\cdot\eta_{2\nu}^2/\xi_1^2\approx 1$ is applied, since only the quantities produced by the fundamental harmonic are considered for the moment: ν_a =1. Factors not important at the moment are eliminated in this way. Thus the formula applied:

$$\frac{M_{synchronous}}{M_{breakdown}} = 2\frac{X_m}{X_s}\frac{\xi_{1\nu_b}}{\mu_a} = 2\frac{X_m}{X_s}\frac{\xi_{1\nu_b}}{\nu_b} \tag{26}$$

$\nu_b = \mu_a$ is as condition of occurring synchronous parasitic torque.

The resulting, residual synchronous parasitic torque in case of *stator* slot skew from (10)

$$\frac{M_{residual}}{M_{breakdwn}} = \frac{M_{synchronous}}{M_{breakdown}}\Delta_{stator} = 2\frac{X_m}{X_s}\frac{\xi_{1\nu_b}}{\nu_b}\frac{1}{\xi_{1\nu_b}}\frac{1}{\nu_b}\frac{3}{\pi} = 2\frac{X_m}{X_s}\frac{1}{\nu_b^2}\frac{3}{\pi} \tag{27}$$

The surprising result is that regardless of q_1, regardless whether $\xi_{1\nu b}$ was a slot-harmonic or not while skewing, the residual synchronous parasitic torque is the same; it depends only from the harmonic order number creating that torque.

The resulting residual synchronous parasitic torque in case of *rotor* slot skew from (24)

$$\frac{M_{residual}}{M_{breakdwn}} = \frac{M_{synchronous}}{M_{breakdown}}\Delta_{rotor} = 2\frac{X_m}{X_s}\frac{\xi_{1\nu_b}}{\nu_b}\frac{\sin(\nu_b\pi/6q_1)}{\nu_b\pi/6q_1} \tag{28}$$

Substitute $\xi_{1\nu b}$

$$\xi_{1\nu_b} = \frac{\sin(\nu_b\pi/6)}{q_1\sin(\nu_b\pi/6q_1)} \tag{29}$$

$$\frac{M_{residual}}{M_{breakdown}} = \frac{M_{synchronous}}{M_{breakdown}}\Delta_{rotor} = 2\frac{X_m}{X_s}\frac{1}{\nu_b}\frac{\sin(\nu_b\pi/6)}{q_1\sin(\nu_b\pi/6q_1)}\frac{\sin(\nu_b\pi/6q_1)}{\nu_b\pi/6q_1} =$$

$$= 2\frac{X_m}{X_s}\frac{1}{\nu_b}\frac{\sin(\nu_b\pi/6)}{\nu_b\pi/6} \tag{30}$$

It is not difficult to notice that the last term of (30) is identical to (10). Finally

$$\frac{M_{residual}}{M_{breakdown}} = 2\frac{X_m}{X_s}\frac{1}{\nu_b^2}\frac{3}{\pi} \tag{31}$$

Both (27) and (31) result in the outcome being independent of q_1 and q_2' and whether $\xi_{1\nu b}$ was a slot-harmonic or not while skewing; the result depends only from the harmonic order number creating that torque. Furthermore, (27) and (31) give the same result, although the formulas of the reduction factors for the stator and rotor skewing are different. It has now been proven with formula that the skewing by one stator slot pitch, whether it is performed on the stator or the rotor, gives the same result, the two skews are equivalent. Formulating as theorem:

Skewing by one stator slot pitch results in the same residual synchronous parasitic torque with q_1 = integer, full pitch stator winding regardless whether the skew is performed on the stator or the rotor, the two skews are equivalent.

The value of the remaining, residual synchronous parasitic torque is always the same, it is independent of the (relative) number of slots of both on the stator and on the rotor, as well as independent of the winding factor of the stator harmonic participating in the generation of that torque (thus whether it is slot harmonic or not). It depends only from the order number of harmonics participating in the generation of the synchronous parasitic torque. However, since that harmonic order number depends solely on the rotor slot number per pole, not on the combination of the stator and rotor slot numbers, the whole phenomenon depends solely on the rotor slot number at the end.

Note that the above formulation does not mean that the stator and rotor skews are not equivalent for the rest of stator windings that means for those other than integer q_1 full pitch winding; it merely means that the present derivation proves it only for this particular case.

It shall be remarked that when calculating the values with slot skew the modification of the leakage reactance (see Section 9. later) is not considered at the moment.

6.2. Application of the rotor slot skew in practice

Looking at the effect of skewing, that is, at the number of rotor slots which is worth or even necessary to skew, let us examine whether it is possible to set up a "rule of thumb".

As an illustration, consider [11] Table VI, which deals with a wide range of number of slots in four poles; data are taken from there.

For such slot numbers and torque-order numbers, where only the slot harmonics of the stator appear, a significant reduction occurs. These are rotor slot numbers that create synchronous parasitic torque only in standstill: q_1=q_2', q_2'=integer, and the so-called half-slot numbers, for which q_2'=q_2+1/2 (where q_2 is a simple positive integer); in general, where the denominator is neither equal to the phase number of the stator nor its integer multiple [11]. For these, skewing yields a significant result; however, the initial synchronous parasitic torque is by far the highest just for these slot numbers. These will be investigated in the next section.

Now, corresponding to practice, which practice in not really understandable for the author, the q_2'=q+1/3 (so-called third slot) dangerous rotor slot numbers will be investigated; they are dangerous because the synchronous parasitic torque created by the lowest order harmonic occurs in the motor range [11].

Consider slot number Z_2=28 as an example. Acc. to [11] the critical harmonic is ν=13 .

At slots 24/28 [11], ν=13 is just the stator slot harmonic, which the skewing has significantly reduced, but the question is whether this is sufficient. By calculation (β=π/12):

to $\mu_a = -13$ $$\frac{\sin 13\pi/12}{13\pi/12} \approx \frac{0.26}{\pi} = 0.08$$

Residual synchronous parasitic torque [11]:

$2.27 \cdot 0.08 = 0.18\ M_{break}$.

Further a significant torque is generated at standstill by substituting e=±6 by ν_b = -83 - 85: 0.7 M_{break} which will be reduced by skewing to ≈(1/4)/7π= 0.008 M_{break} , to practically zero, thanks to that harmonic being very high.

At the inexplicably popular 36/28 slot number, although the torque in the motor range is created by the stator non-slot-harmonic, it is still dangerous but the effect of the skewing is limited. The effect of skew on the torques created by the lowest harmonics (β=π/18):

$\mu_a = -13$ $$\frac{\sin 13\pi/18}{13\pi/18} = 0.33$$ is moderate.

Without skewing [11] $M_{synchr}/M_{break} = 0.51$
With skewing: $0.51 \cdot 0.33 = 0.17\ M_{break}$

$\mu_a = 29$ $$\frac{\sin 29\pi/18}{29\pi/18} = 0.185$$

$\mu_a = -55$ $$\frac{\sin 55\pi/18}{55\pi/18} = 0.018$$ $\mu_a = 71$ $$\frac{\sin 71/18}{71\pi/18} = 0.014$$

The following comments can be made about the calculation:

- in this example only (against the general rule), significant standstill torque is not created

- it is questionable whether the reduction in the parasitic torque occurring in the motoric range of the smallest order number is sufficient

- there is enough to deal with order numbers up to twice the number of slots per pole pair of the stator

- it can be seen from the formula that if the denominator in the sin function would be 14 instead of 18, both remaining low order synchronous torques would disappear. By calculation (β=π/14):

$$\frac{\sin 13\pi/14}{13\pi/14} \approx \frac{0.225}{\pi} = 0.072\,.$$

Residual synchronous parasitic torque:

$M_{synchr}/M_{break} = 0.51 \cdot 0.072 = 0.036$; it is an effective skewing.

It is not difficult to notice that this is just skewing by one *rotor* slot pitch, as shown in Figure 10d. This is not surprising because both low order harmonics originate from the rotor slot number.

The 48/28 slot number, which is of theoretical interest only, is also better to skew by one rotor slot pitch.

Returning back to 24/28 machine, the reduction factor with stator slot skew (β=π/12) or with rotor slot skew (β=π/14) is almost the same. In case of $\beta=\pi/13=\pi/\nu_{b\text{-critical}}$, however, the synchronous torque drops to mathematical zero. If Möller had skewed at this slot number not according to the stator slot pitch, but according to the critical harmonic, he would have obtained a mathematical zero torque. The resulting noise component would also be mathematically zero. Such consideration has relevance only in case of synchronous torque in rotation. In case of synchronous torque in standstill, it is skewed always against *two* harmonics $\nu=2mq_1\pm1$ expediently by $2mq_1$.

This arrangement is a good example of how the skew can be used in a really targeted manner. It indicates that *the stator slot harmonic and/or the skewing by one stator slot pitch does not play such an exclusive role* as is usually attributed to it.

Table 3. shows the effect of skewing the rotor by one rotor slot pitch acc. to (24). It is clear that such skewing effectively reduces just the rotor slot harmonics.

Table 3: Reduction Factors of Harmonics Produced by the Rotor by Skewing the Rotor by one Rotor Slot Pitch

q_2'	2 1/3	2 2/3	3 1/3	3 2/3
β	π/14	π/16	π/20	π/22
harmonic	reduction factor			
ν	Δ	Δ	Δ	Δ
5	0,803	0,847	0,900	0,917
7	0,637	0,714	0,810	0,842
11	0,253	0,385	0,572	0,637
13	0,076	0,218	0,436	0,517
17	-0,163	-0,058	0,170	0,270
19	-0,211	-0,149	0,052	0,153
23	-0,175	-0,217	-0,126	-0,043
25	-0,111	-0,200	-0,180	-0,116
29	0,034	-0,098	-0,217	-0,203
31	0,090	-0,032	-0,203	-0,217

Based on this, the following theorem can be established:

- *if the dangerous synchronous parasitic torque is created by ν_b that is a stator slot harmonic, then skewing by one stator slot pitch must be applied, if ν_b is not a stator slot harmonic (but obviously a rotor slot harmonic) then skewing by one rotor slot pitch must be done.*
- *further: if the synchronous parasitic torque is generated in standstill, it is created always by stator slot harmonics, therefore skewing with a stator slot pitch must be used. If the synchronous torque is generated in rotation, it shall be proceeded according to the provisions of the previous paragraph. In this case, however, it is possible to skew neither acc. to stator slot pitch nor acc. to rotor slot pitch but according to the critical harmonic to be suppressed and then the resulting torque is mathematically zero.*

Analysis like in [11] Table VI. is therefore indispensable.

It should be noted that the result for the slot harmonic is *sensitive to the accuracy of skew* compared to the theoretical one, corresponding to the variation of the sin function near π. For example, if the critical harmonic is ν=13 and the theoretical skew is β=π/12, then a slight difference in the actual skew might lead to a change in a ratio of up to 1:2. Calculation for the rotor slot skew

$$\frac{\sin 13\pi/11}{13\pi/11} = 0.15 \qquad \frac{\sin 13\pi/12}{13\pi/12} = 0.08 \qquad \frac{\sin 13\pi/13}{13\pi/13} = 0$$

With the mathematical proof, the differential $d\Delta_{rotor}/d\beta$ of (24) must be formed. Omitting the details of the mathematical analysis, the differential near $\nu\beta\approx\pi$ is $\approx 1/\beta=2m_1q_2'/\pi$. In the example: the appr. value is: $1/\beta = 3.82$; the exact value of the differential is: 3.67; it is very high. The sensitivity increases linearly with q_2'.

The investigated sensitivity has significance only in the case of the synchronous torque in rotation, where it is skewed against a single harmonic. In the case of synchronous torque in standstill, it is skewed by $2mq_1$ against two harmonics, $\nu=2mq_1+1$ and $\nu=2mq_1-1$, so a slight inaccuracy in the skewing is irrelevant.

7. Summation of Synchronous Parasitic Torques Occurring in Standstill

This chapter is a further development and partially *fundamental* modification of Chapter E 2 of [11].

Let us take the slot layout 36/36 and 24/24 as example; they are also frequently investigated by researchers.

The need to sum synchronous parasitic torques in standstill arises only for those rotor slot numbers that produce synchronous parasitic torque only in standstill; however, in this case, summation is always necessary. The rotor slot number is expressed in the usual way: $q_2'=q_2+p/r$, where q_2 is a positive integer, p and r are positive integers, and p≤r. If r is not equal to the stator phase number or an integer multiple thereof, the need for summation always arises [11].

In such a case it is a matter of summing *spatial* torque components, which are expressed vectorially:

$$M = e^{j\delta} + \frac{1}{2}e^{j2\delta} + \frac{1}{3}e^{j3\delta} + \frac{1}{4}e^{j4\delta} + \frac{1}{5}e^{j5\delta} + \frac{1}{6}e^{j6\delta} + \dots \quad (32a)$$

On trigonometric way:

$$M = \sin\delta + \frac{1}{2}\sin 2\delta + \frac{1}{3}\sin 3\delta + \frac{1}{4}\sin 4\delta + \frac{1}{5}\sin 5\delta + \frac{1}{6}\sin 6\delta \dots \quad (32b)$$

δ is the angular position of the rotor related to one rotor slot pitch.

We are looking for the maximum of this.

This infinite series is the Fourier-solution of the function

$$f(x) = \frac{\pi - x}{2}\ ,\ 0<x<2\pi\ ,\ f(0)=0 \quad (33)$$

Here x stands for δ.

The maximum of the infinite series is π/2 times the first, largest torque component and occurs at x=0. However, the maximum of the finite series can be higher, as shown in Figure 11.

This series acc. to (32a) and (32b) are typical of the 36/36, 24/48 and 24/18 slot arrangements.

It can be seen from (32a) and (32b) that the magnitude of the higher order spatial harmonics decreases only "slowly," the higher orders cannot be neglected in any way; therefore, their exact summation is of fundamental importance. At the same time, we also saw in [10] that one should not go to infinite summation.

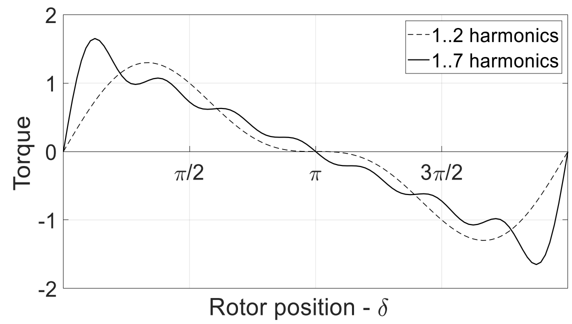


Figure 11: Synchronous parasitic torque occurring at standstill as a function of the angular position of the rotor for 36/36 slots

We performed our calculations for Figure 11. and Figure 12 for the harmonic order up to $\nu_b \leq 150$ [11]. Correspondingly, the first seven torque components for the 36/36 slot number and the first ten torque components for the 24/24 slot number must be included in the calculation.

A representation of the line with the first seven torque components is shown in Figure 11.

The value of the maximum torque is 1,67 times the value of the first torque component, which occurs at δ=0.34≈π/9.

In the other case, the sign of the components alternates, that of the even components is negative, this is typical of 24/24, 48/48, 24/30 and other slot numbers.

The series is:

$$M = \sin\delta - \frac{1}{2}\sin 2\delta + \frac{1}{3}\sin 3\delta - \frac{1}{4}\sin 4\delta + \frac{1}{5}\sin 5\delta - \frac{1}{6}\sin 6\delta \dots \quad (34)$$

This infinite series is the Fourier-solution of the function

$$f(x)=x/2 \qquad -\pi< x <\pi\ ,\ f(\pi)=0 \quad (35)$$

The maximum of the infinite series is again π/2 times the first, the largest torque and occurs at x=π.

A representation of the line with the first ten torque components is shown in Figure 12.

The value of the maximum torque is 1,71 times the value of the first torque component and occurs at δ=2.86≈8/9π.

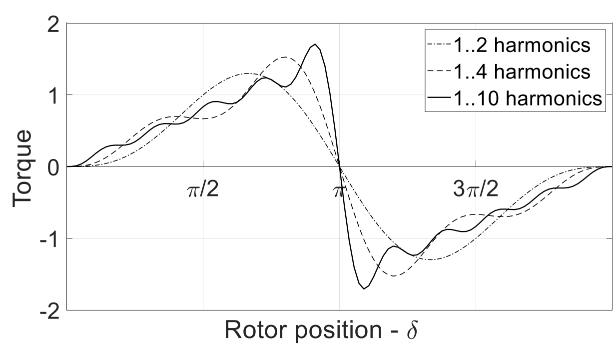


Figure 12: Synchronous parasitic torque occurring at standstill as a function of the angular position of the rotor for 24/24 slots

The effect of skew is calculated by multiplying each torque component as a function of the angular position of the rotor by the corresponding skew reduction factor Δ, and then summing them up. Because the skew affects each component differently, the final result cannot be included in a formula, but a manual summation would be required. It is certain here that the summation of not 7 or 10 waves, but rather only 2-3 waves may become necessary; the number 2 wave will be stronger reduced, and that of number 3 will be much stronger; therefore, not much error will be made if the summation is actually dispensed with for a skewed machine. The decrease of the first, highest torque component:

for q_1=q_2'=2 $\frac{\sin 11\pi/12}{11\pi/12} \approx \frac{1/4}{\pi} = 0.08$

for q_1=q_2'=3 $\frac{\sin 17\pi/18}{17\pi/18} \approx \frac{1/6}{\pi} = 0.053$

for q_1=q_2'=4 $\frac{\sin 23\pi/24}{23\pi/24} \approx \frac{1/8}{\pi} = 0.04$

The synchronous parasitic torque remaining after skewing [11]

for q_1=q_2'=2 4.92·0,08 = 0.4 M_{break}
for q_1=q_2'=3 3.31·0.053 = 0.175 M_{break}
for q_1=q_2'=4 1.3 ·0.04 = 0.05 M_{break}

It should be noted here that the so-called “half slot” numbers (q_2'=q_2±1/2) quite rightly escape the attention of designers. With slot skew, however, for example, at 24/30 slot number, the remaining synchronous parasitic torque in standstill is [11]: ≈0.99·(1/4)/(5π)=0.016 M_{break} practically zero. While a straight rotor slot is out of question, skewing makes this slot number usable without problems if skewed by one stator slot pitch. In addition, for 36/30 and 48/30 slot numbers, too. We therefore encourage designers to use this arrangement.

8. Calculation of the Change of the Leakage Reactance of the Machine

In addition to the previous phenomena, another change takes place in the machine, as the reactance of the machine also change, in a well-known way. From this point of view, the change in leakage reactance has a greater effect, although minor changes in the fundamental magnetization reactance can also be experienced. Let us calculate the effect of this change for the machine typically characterized by X_m=3, X_s= 0.2 ($I_{starting}$=5·I_{rated}), and X_m/X_s = 15, used in all our studies so far.

The winding factor of the skew of the rotor with one stator slot pitch is obtained from (4) for q_1 =q_2'= 3 [14]

$$\xi_{skew} = 1-0.41 \cdot (1/3q_1)^2 = 0.995 \quad (36)$$

the leakage coefficient of the skew

$$\sigma_{skew} = 2\,(1-\xi_{skew}) = 0.0102 \quad (37)$$

excess leakage reactance

$$X_s = X_m \cdot \sigma_{skew} = 0.0306 \quad (38)$$

the rate of change of the leakage reactance:

$$x_{s\Delta} = 0.2 / 0.23 = 0.87 \quad (39)$$

The machine's data changed as a result of skewing:

X_m = 2.97 X_s = 0.23 X_m / X_s = 12.91

The ratio of reactance instead of 15 for the skewed machine is 12.91, the ratio of change

$$x_{\Delta} = 12.91/15 = 0.86 \quad (40)$$

The synchronous parasitic torque relative to the actual breakdown torque decreased in proportion to x_Δ. Note that the calculations in Sections 4. and 6. do not contain this factor at the moment. The breakdown torque of the machine also decreased, in proportion to $x_{s\Delta}$, this is the "price to be paid" or “trade-off” for the advantage of skewing what designer need to consider. The absolute value of the synchronous parasitic torque decreases in proportion to the product of $x_{s\Delta} \cdot x_\Delta$.

However, this value is deliberately not calculated now. The reduction of the synchronous parasitic torque and the breakdown torque of the machine in the same proportion cannot be considered as a useful consequence of skewing; on the contrary, it specifically limits the extent of skewing.

9. The Effect of Skewing on the Asynchronous Parasitic Torques

The relevant part of the measurement series carried out by Möller [16] citied already by us before is repeated here on Figure 13. One rotor skewed by one stator slot pitch was also manufactured and measured for one of the 19 rotors examined by Möller.

A comparison of the measurements 24/28 slots with straight and skewed rotor slots shows the expected reduction in the synchronous parasitic torque.

However, another important phenomenon has also emerged. When measuring the straight-slot rotor (Z_2>Z_1 in this case), significant asynchronous torques were experienced, namely, by the low-order 5th and 7th harmonics and the 11th and 13th harmonics being stator slot harmonics. However, asynchronous parasitic torques practically disappear in the skewed machine. Therefore subject measurement represents strong evidence for us regarding the complete explanation; it validates the theory provided by us.

In the case of a skewed bar, the voltage and its phase position induced in the rotor bar, while moving along the bar, describe a complete circle, which, in the case of the 12th harmonic, reaches the end of the bar covering just 360° and returns to the initial position. The 11th harmonic only travels 330°, so its original value is reduced to 1/11 of the chord/circumference arc ratio, that is, its inducing effect practically disappears. Additionally, to 1/13 of the 13th harmonic. This phenomenon has also reduced the inducing effect of the 7th and 5th harmonics, ≈2/7 and ≈2/5 in the chord/arc proportion. It is of basic importance that the disappearance of the asynchronous torque also means that an attenuation effect on the concerned harmonics does not occur. Therefore, the 13th harmonic of the stator, remaining in magnitude, was still able to create a high synchronous parasitic torque. It was the 13th harmonic of the *rotor*, however, which was decreased by skewing and caused a reduction in the synchronous parasitic torque at the end. *This is the complete physic, the complete effect of the rotor skewing.*

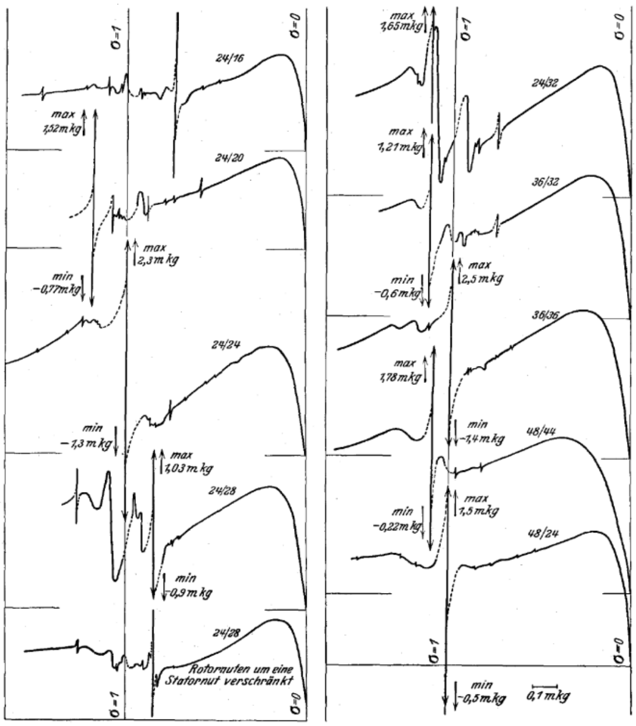


Figure 13: Highest synchronous torques, the measured values are given in mkg see Möller [16] Figure 8.

The rotor slot skewing, in accordance with the true physical background, did not eliminate the synchronous parasitic torque but eliminated the asynchronous parasitic torque; the synchronous parasitic torque was only reduced.

Now, a formula for calculating the exact value will be derived. To do this, the changes in the elements of an arbitrary harmonic circuit, as shown in Figure 2 is examined.

Definition of the skew factor of the rotor slot acc. to (14) ([12] (206) and (268a)) again

$$\chi_{2\nu} = \frac{\sin(\nu \frac{s}{\tau_{h2}} \frac{p\pi}{Z_2})}{\nu \frac{s}{\tau_{h2}} \frac{p\pi}{Z_2}} \tag{41}$$

Substitute the β value corresponding to the skew of a rotor with one rotor slot pitch in (7): $s/\tau_{h2}=1$, $\beta=\pi/2mq_2'=\pi/(Z_2/p)$. We obtained the unsurprising result that formula (41) based on physical considerations and formula (7) obtained from Fourier analysis are identical.

Note that if $s=\tau_{h2}$, i.e. we skew with exactly one rotor slot pitch, then (41) is the same as the definition of $\eta_{2\nu}$ acc. to (2), i.e. $\chi_{2\nu}$ is equal to $\eta_{2\nu}$. This does not originate from their physical content, but later, in the evaluation of (51), we make use of this identity.

The derivation will follow the way, used first by us, in [9], by inserting $\chi_{2\nu}$ into the assigned formulas.

The main field reactance of the fundamental harmonic changes linearly with the skew factor $X_{m\chi}=X_m \cdot \chi_{2,\nu=1}$. The $X_{m\nu}$ reactance, which means the "main field" reactance for the harmonic circuit, is

$$X_{m\nu\chi} = X_{m\nu} \cdot \chi_{2\nu} \tag{42}$$

accordingly.

The complete definition of the differential leakage coefficient ([12] (268))

$$\sigma_{2\sigma\nu\chi} = \frac{1}{\chi_{2\nu}^2 \eta_{2\nu}^2} - 1 \tag{43}$$

The differential leakage

$$X_{s\sigma 2} = (\frac{1}{\chi_{2\nu}^2 \eta_{2\nu}^2} - 1) X_{m\nu} \chi_{2\nu} \tag{44}$$

The total reactance of the harmonic circuit

$$X_{m\nu}\chi_{2\nu} + (\frac{1}{\chi_{2\nu}^2 \eta_{2\nu}^2} - 1) X_{m\nu}\chi_{2\nu} = \ldots = \frac{X_{m\nu}}{\chi_{2\nu}\eta_{2\nu}^2} \tag{45}$$

differs from the original value in the proportion $1/\chi_{2\nu}$. "Main field" reactance decreased, leakage reactance increased.

Again, in harmonic circuits, only the differential leakage reactance is included as the leakage reactance; the rest of reactance are negligible [12], [15]. To calculate the harmonic breakdown slip, the change in resistance must also be taken into account, since the resistance reduction factor of the unchanged rotor resistance changes in the ratio $1/\chi^2$ ([12] (170) p. 100). The harmonic breakdown slip

$$s_{b\nu\chi} = \frac{R_{2\nu}/\chi_{2\nu}^2}{(X_{2m\nu} + X_{2\sigma\nu})/\chi_{2\nu}} = \frac{R_2}{X_m} \nu^2 \eta_{2\nu}^2 \frac{1}{\chi_{2\nu}} = \frac{s_{b\nu}}{\chi_{2\nu}} \tag{46}$$

$s_{b\nu}$ changes, the scale of slip too.

Acc. to voltage equation supplemented by $\chi_{2\nu}$ accordingly:

$$I_{2\nu} = -j \frac{s_\nu X_{2m\nu}\chi_{2\nu}}{R_{2\nu}/\chi_{2\nu}^2 + js_\nu (X_{2m\nu} + X_{2\sigma\nu})/\chi_{2\nu}} I_1 \tag{47}$$

The endpoint of $I_{2\nu}$ describes a circle as a function of s_ν, as shown in Figure 14. but the scale is different due to $\chi_{2\nu}$.

Arranged and substituted

$$I_{2\nu} = -j \frac{s_\nu \chi_{2\nu} \cdot \eta_{2\nu}^2}{s_{b\nu\chi} + js_\nu} I_1 = \ldots = (-\frac{s_\nu^2 \chi_{2\nu} \cdot \eta_{2\nu}^2}{s_{b\nu\chi}^2 + s_\nu^2} - j \frac{s_\nu s_{b\nu\chi} \chi_{2\nu} \cdot \eta_{2\nu}^2}{s_{b\nu\chi}^2 + s_\nu^2}) I_1 \tag{48}$$

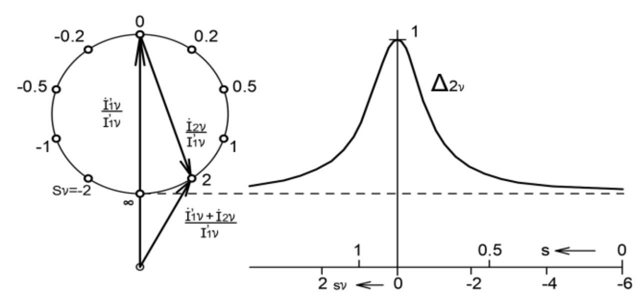


Figure 14: Vector diagram of the currents of the harmonic circuit ν=7 and the attenuation factor $\Delta_{2\nu}$ belonging to this circuit (=to this harmonic) (see [12] Richter Vol. IV p. 150, Figure 106)

The imaginary component creates the torque. Substituting $s_{b\nu\chi}$ acc. to (46) then $s_\nu = s_{b\nu\chi}$ yields

$$I_{2\nu} = \ldots - j\frac{\chi_{2\nu}\cdot\eta_{2\nu}^2}{2}I_1 \quad (49)$$

changed in proportion $\chi_{2\nu}$.

The maximum power

$$P_{\max\chi} = 3\cdot U_{1\nu}\cdot I_{2\nu} = 3\cdot I_1 jX_{m\nu}\chi_{2\nu}(-j\frac{\chi_{2\nu}\eta_{2\nu}^2}{2})I_1 = P_{\max}\chi_{2\nu}^2 \quad (50)$$

changed by the square of $\chi_{2\nu}$.

The change the asynchronous breakdown torque is therefore

$$\frac{M_{b\nu\chi}}{M_b} = \frac{X_m}{X_s}\frac{\xi_\nu^2}{\xi_1^2}\frac{\eta_{2\nu}^2}{\nu}\chi_{2\nu}^2 = M_{b\nu}\chi_{2\nu}^2 \quad (51)$$

The formula correctly shows the effect of the factors ξ_ν^2, ξ_1^2, $\eta_{2\nu}^2$ and $\chi_{2\nu}^2$ on the harmonic torque of ν, in accordance with the physical picture. The effect, including the quadratic effect of slot skew, is consistent with [12] (293) p. 180.

The result would have been accurate (slightly smaller) if $(I_1+I_2)\cdot jX_{m\nu}$, the absolute value of the resulting current, was substituted instead of $I_1\cdot jX_{m\nu}$ in (50). Then, it would have been taken into account that some attenuation already occurs on slip $s_{b\nu}$, instead of no attenuation on slip $s_\nu=0$.

The rate of reduction is also illustrated by Figure 4 although it was developed for $\eta_{2\nu}^2$. If skewed by one rotor slot pitch then $\chi_{2\nu}^2=\eta_{2\nu}^2$ and the ratio in Figure 4. corresponds to $\chi_{2\nu}^2$; if skewed by one stator slot pitch, then approximately by that amount.

Skewing of the rotor slot effectively reduces the asynchronous parasitic torque, as it was invented just for this purpose. The comment on Figure 13 is now also confirmed by the formulas. From the comparison of (15) and (51) comes out that the slot skew affects the synchronous parasitic torque and the asynchronous parasitic torque in different manner: it reduces the former linearly, the latter by its square: corresponding to the difference in their physics. Another difference is that (in the case of $Z_2>Z_1$) the critical harmonics in both cases do not necessarily coincide because they are independent from each other.

Skewing the rotor slot significantly reduces the response of the rotor to all stator harmonics in this way except fundamental harmonic. Therefore, in skewed machines, there is another reason to calculate the rotor harmonics created only by the fundamental harmonic stator MMF when calculating the synchronous parasitic torque and radial magnetic force waves. However, they must be taken into account in their original value in the role of ν_b because the harmonic attenuation of all stator harmonics is naturally greatly reduced and practically eliminated.

10. Effect of Rotor Slot Skewing on the Attenuation of Stator Differential Leakage

Attenuation is related to the asynchronous parasitic torque; therefore, they must be treated together as it was done in [9].

It is observed in the previous chapter that the harmonic current has decreased as a result of skewing, so the stator differential leakage will be attenuated to a lesser extent. Complete attenuation factor [12] (269)

$$\Delta_\nu = 1 - \chi_{2\nu}^2\eta_{2\nu}^2 \quad (52)$$

came closer to 1.

In this way, the differential leakage is attenuated by

$$\sum_{\nu\neq 1}^{2mq_1+1}\chi_{2\nu}^2\eta_{2\nu}^2\frac{1}{\nu^2}\frac{\xi_\nu^2}{\xi_1^2}X_m \quad (53)$$

This implies that this value will be smaller. Therefore, the elements of differential leakage affected by attenuation must be calculated according to

$$\sum_{\nu\neq 1}^{2mq_1+1}\frac{1-\chi_{2\nu}^2\eta_{2\nu}^2}{\nu^2}\frac{\xi_\nu^2}{\xi_1^2}X_m \quad (54)$$

After rearranging, we arrive at the definition of the attenuation factor, acc. to Richter [12]:

$$\Delta = 1 - \frac{1}{\sigma_1}\sum_{\nu\neq 1}^{2mq_1+1}\chi_{2\nu}^2\eta_{2\nu}^2\frac{1}{\nu^2}\frac{\xi_\nu^2}{\xi_1^2} \quad (55)$$

where as definition

$$\sigma_1 = \sum_{\nu\neq 1}^{\infty}\frac{1}{\nu^2}\frac{\xi_\nu^2}{\xi_1^2}$$

In [9], we calculated the attenuation factors for the often-used slot numbers, taking also chording into account; the latter was missing in the literature until then. Now, we also consider the effect of skew, for different chording, in Figure 15. The characteristic curves calculated in [9] are repeated, and then the new curves are added next to them to facilitate the comparison.

As expected, the figures show that the attenuation values came very close to 1, especially when $q_2'<q_1$. In these cases, a value of Δ=1 can be assumed, which means that there is no attenuation, the original, unchanged value of the stator differential leakage must be considered. If $q_2'>q_1$, the characteristic curve provides precise guidance on the attenuation to be considered. If $q_1\geq 4$, slot skewing has practically no influence.

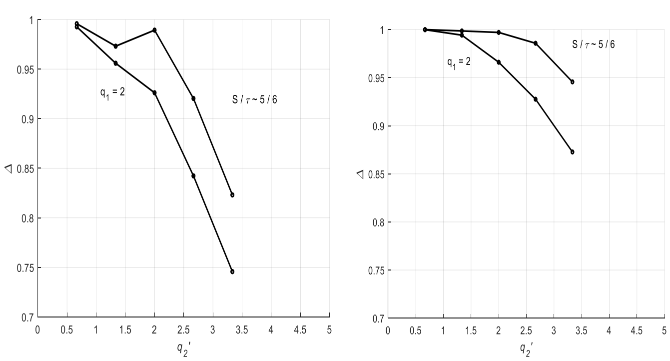


Figure 15a: Characteristic of Δ attenuation factors acc. to Richter ([12], p. 155. Figure 108) modified by the Author, for q_1=2, with no chording and with a chording of S/τ~5/6 , with no skewing, with skewing by one rotor slot pitch

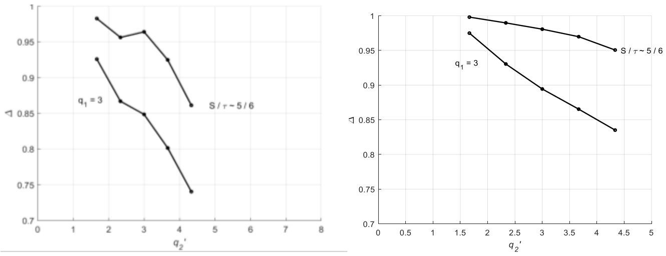


Figure 15 b: Characteristic of Δ attenuation factors acc. to Richter ([12], p. 155. Figure 108) modified by the Author, for q_1=3, with no chording and with a chording of S/τ~5/6 , with no skewing, with skewing by one rotor slot pitch

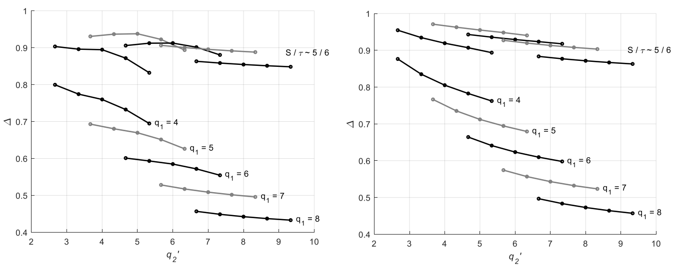


Figure15.c: Characteristic of Δ attenuation factors acc. to Richter ([12], p. 155. Figure 108) modified by the Author, for q_1=4-8, with no chording and with a chording of S/τ~5/6 , with no skewing, with skewing by one rotor slot pitch

11. High Voltage Stator Winding with Open Slots

The formula for the MMF harmonics generated by the stator winding and their winding factor is valid for the theoretical machine with no slots on the stator and rotor, with the conductors of infinitely small width so the slot opening was not taken into account. However, the high-voltage winding is placed in open slots according to standard technology. The circumferential change of the MMF along the slot opening is not stepwise anymore as it is shown in Figure 16.

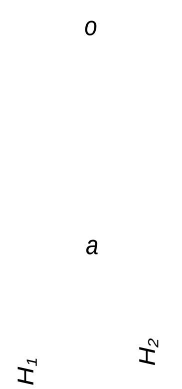


Figure 16: Sketch for the MMF of a conductor with a finite width along the slot opening ([15] Figure 19, p. 46)

Similar conditions also develop in the case of strong saturation of the tooth head.

The width of the slot depends on the design of the machine, within which the width of the copper wire depends on the voltage level and the resulting insulation thickness. From the point of view of the present investigation, it is considered a good general approximation if the width of the conductor is taken as 1/3 of the stator slot pitch. Figure 17 shows the resulting MMF curve for q_1=2 and q_1=3.

Figure 17: Image of MMF of one phase for q_1=2 and q_1=3

The figure is identical to the MMF generated in the case of a stator slot skew of 1/3 stator slot pitch; therefore, a high-voltage stator in itself is equivalent to a stator slot skew of this magnitude.

About the phenomenon, Jordan proved how it reduces the differential leakage; the result is described in [15]. Differential leakage is a general, cumulative quantity. Here, however, we cannot be satisfied with the general evaluation, since the effect on the synchronous torques and on the noise components must be precisely determined one by one. Therefore, the considerations of Chapter 4. must be applied, which requires the analysis of Figure 17.

Figure 17, on the other hand, does not correspond to Figure 9, therefore (11) is not valid. The arrangement can only be examined with a full Fourier analysis. Based on the figure, it is expected that the arrangement corresponding to such a small amount of skew will not have a perceivable influence on the behavior of the machine; however, this will only be partially true. The investigation provides the following result see Table 4.

Table 4: Stator Harmonics in High-Voltage Motors

q_1	2		3
harmonic	magnitude of harmonic		
ν			
5-7	0,24		0,19
11-13	0,80		0,18
17-19	0,17		0,80
23-25	0,42		0,15
29-31	≈0		≈0
35-37	≈0		0,40
41-43	≈0		≈0
47-49	0,20		≈0
53-55	≈0		≈0
59-61	0,2		≈0
65-67	≈0		≈0
71-73	≈0		0,2

Evaluation of it is as follows:

- in order to evaluate the results, the calculated values shall be compared with the winding factor for the respective harmonic. The reason for this is that the values are not reducing factors, but residual values. It can be seen that there is no perceptible change up to the 2nd slot harmonic, which is then reduced by half. The higher harmonics are significantly reduced even by such a small skewing. Still, the design engineer does the right thing if he ignores the effect of open slots in the vast majority of cases.

- it was already shown in [11] III. F that harmonics of higher order, depending on the size of the air gap, do not play a role from the point of view of subject examination. Table 4 shows further that for machines with open stator slots, even the harmonics with order number higher than 2nd slot harmonic have no role except slot harmonics, but they also appear only strongly reduced. This means that in high voltage motors much less harmonics shall be considered as in low voltage ones.

- based on [11] Table VI. 4 pole, however, it can be seen that there might still be one or two slot numbers that could be used in high-voltage machines, such as the Z_1/Z_2=24/32 slot number; it is a so called "of 2/3q" rotor slot number, the dangerous highest torque occurs in brake range, the second high torque in motoric range is acceptable, the high stillstand torque is caused by high order harmonics, therefore, it is fairly reduced by open stator slot

- at this point the variation in magnetic conductivity of the airgap cannot remain out of consideration; [10] provides detailed guidance in the matter.

12. Summary

In this study, the effect of the skew of the rotor slot on the parasitic torques of a squirrel-cage induction motor was examined through analytical method. It was established that the generally accepted physical explanation for this phenomenon is incomplete and therefore misleading it does not cover the complete physical truth.

Here, the complete physic was studied; a clear distinction was made between the physics of the effect of skewing on synchronous and asynchronous parasitic torques. Further, a clear distinction was made between the physics of stator slot skew and rotor slot skew. The explanation of rotor slot skew regarding synchronous parasitic torque is: the skewing reduces the harmonics of the fundamental harmonic current of the rotor, thereby reducing the synchronous parasitic torque. A modified theorem defining the creation of synchronous parasitic torques is proposed. The calculation was based on comparing the stepped MMF curve of the straight rotor slot and the trapezoidal MMF curve of the skewed slot. By setting the harmonic content of the two curves in proportion, a new formula was provided.

A new basic theorem was formulated: the magnitude of the residual synchronous parasitic torque due to skewing by one stator slot pitch is always the same, being independent from the magnitude before skewing, from the slot number combination and from the winding factor of the stator harmonic creating the torque; it is independent from whether the skew is performed on the stator or on the rotor; it depends only from the order number of the actual harmonic: all of these are proven by formula.

The degree of skewing on the rotor is not determined a priori, as it was thought until now, but can be applied on a targeted manner with regard to the dangerous harmonic torque; another theorem was formulated on when to skew according to the stator and when acc. to the rotor slot pitch or with other pitch. Further, the theorem was supplemented in case of synchronous parasitic torque in rotation: the skewing may be done acc. to critical harmonic in order to achieve mathematical zero result. With this theorem, the explanation of the effect exclusively by the stator slot harmonic as well as preforming the skewing exclusively by one stator slot pitch is exceeded.

It was proved in [10] at the first time that the synchronous parasitic torque and radial magnetic force can be transferred to each other; that is, the results can also be used in noise reduction. It is a task for further research to investigate the distinct effect of skew on the noise because the topic is generally not researched in sufficient depth.

A new formula is provided for the calculation of effect on the asynchronous parasitic torque, too. The slot skew affects the synchronous parasitic torque and the asynchronous parasitic torque in different manner: the skew factor reduces the former linearly, the latter stronger, by its square.

The effect of the skewing on the stator differential leakage was thoroughly calculated for a wide range corresponding to practice. The differential leakage is (much) less attenuated in a skewed slot machine.

Based on its physics also the open stator slot of a high-voltage machine belongs to this topic, there are one or two slot numbers that might become usable in high-voltage machines although being forbidden for low voltage ones.

The above summarized results should indicate the direction to be followed for further research in the case of using more advanced methods by including neglected aspects and then performing measurements.

The contribution of this study is that analytical approach was chosen namely Fourier analysis of the MMF of the skewed machine instead of other methods giving photo-like results only. The paper revealed the basic laws of skewing, this way of analysis made it possible to discover key theorems.

The conclusion is that the design engineer should not apply rotor slot skewing always on the same way but purposefully, on a targeted manner; with torque in rotation, he can completely eliminate (mathematical zero) the critical torque and noise component. Reduction of breakdown torque, however, as "price to be paid" for the advantages need to be considered as it has been done so far.

With present study, the entire scope of the topic is covered.

13. Appendix

13.1 MMF harmonics

The basic formulas for the order number of space harmonics:

$$\nu_a = 6g_1 + 1 \quad (56a)$$
$$\mu_a = e\cdot Z_2/p + \nu_a = e\cdot 2mq_2' + \nu_a \quad (56b)$$
$$\nu_b = 6g_2 + 1 \quad (57)$$

The rotor harmonics generated by the fundamental harmonic:

$$\nu_a = 1 \qquad \mu_a = e\cdot Z_2/p + 1 = e\cdot 2mq_2' + 1 \quad (58)$$

The harmonics of the stator MMF called here ν_b are considered now for q_1 =2, together with the response of the rotor

ν_b= -5 $\quad 0<\eta_{2,-5}^2<1$
ν_b= 7 $\quad 0<\eta_{2,7}^2< 1$
ν_b= -11 $\quad \eta_{2,-11}^2 \approx 0$
ν_b= 13 $\quad \eta_{2,13}^2 \approx 0$

The magnitude of these harmonic fields is attenuated due to response of the rotor by the Δ_ν rotor attenuation factor [12] (269) p. 154:

$\nu_b = -5 \quad \xi_5 \quad \xi_5 \cdot \Delta_5 = \xi_5 \cdot (1- \eta_{2,-5}^2)$

$\nu_b = 7 \quad \xi_7 \quad \xi_7 \cdot \Delta_7 = \xi_7 \cdot (1- \eta_{2,7}^2)$

$\nu_b = -11 \quad \xi_{11} = \xi_1 \quad \xi_{11} \cdot \Delta_{11} = \xi_{11} \cdot (1- \eta_{2,-11}^2)\ (= \xi_1)$

$\nu_b = 13 \quad \xi_{13} = \xi_1 \quad \xi_{13} \cdot \Delta_{13} = \xi_{13} \cdot (1- \eta_{2,13}^2)\ (= \xi_1)$

The apparent winding factor of the slot harmonics remains the same, as it generates a very small current in the rotor to attenuate it.

The stator harmonics (56a) and (57), which are both numerically and physically *identical*, are called ν_a or ν_b according to their roles.

13.2 Synchronous parasitic torques in standstill

The synchronous parasitic torque generated in standstill is created by the difference between the torques generated by two *adjacent* harmonics. However, they add up if the two torques have different signs. For this, it is necessary that the winding factors of the two (adjacent) harmonics have the same sign: this condition is only fulfilled in the case of slot harmonics. In other words, a significant torque in standstill is generated only with the contribution of the (stator) slot harmonic; otherwise that torque is practically zero. The torque is therefore significant for two reasons: because both of them are created by slot harmonics and then they are summed up. Only slot skew with a stator slot pitch protects against these; but since it should protect against two different torques at the same time, the resulting torque cannot be mathematically zero (see end of Chapter 7).

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Conflicts of Interest: "The author declares no conflicts of interest."

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