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Direction of Arrival Using 2-D Matrix Pencil Method

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ABSTRACT

In this research work, we proposed a technique to decrease the complexity of 2-D Matrix Pencil (MP) for the direction of arrival (DOA) evaluation by combining two perpendicular arrays: The first one is the uniform linear array (ULA) and the second one is a uniform circular array (UCA). This special structure for the 2D Matrix Pencil pipe to a powerful methodology for real-time implementation on a digital signal processor, while the MP used to correlated and uncorrelated sources with the presence of a white noise. The obtained results show that proposed MP method gives better results at the level of the precision and the estimation of the DOA compared to the published measure.

1. Introduction

The technological progress in the telecommunications field has given a rise to a remarkable increase in systems for the exchange and transport of information through more accessible and easily manageable means: this is the case of wireless systems which have several advantages in particular at the level of their high throughput and their various applications such as: telephony, remote localization, medicine and military applications [1-3].

Smart antennas have evolved to meet the demands of many applications such as mobile, radar and marine applications. Several research studies have been published on this subject for the purpose of treating this new generation of antennas in order to evaluate the direction of arrival (DOA) signals received by antenna networks of different forms such as the Uniform Linear Network (ULA) or the Uniform Circular Network (UCA). To do this, several methods have been proposed so as to evaluate and estimate the directions of arrival of signals have been proposed, for example: MUSIC, ESPRIT, MLE [4] and the Matrix Pencil [5]. In this manuscript, we study the method of the Matrix Pencil comparing it with others studies published.

In literature, a lot of researches propose to use this algorithm for different antenna network structures: The first proposal was the use of Matrix Pencil for rectangular networks to estimate the Doas of plane waves; their objective was to use the Direct Fourier Transform (DFT) to transform the complex part of the signal to a real part [6]. The second proposal was to use the same network, but this time, trough the transformation of MP method while is based on the information collected by the data matrix of the pencil method, for the purpose of reduced the complexity of the calculation and estimate the desired DAO [7]. The third proposal was the use of the number of samples as the den to apply to the Matrix Pencil in order to extract the DOA. So, our proposal is to combine the ULA structure with the UCA, first, to benefit from the advantages of each network and to compensate for these disadvantages, then to minimize the complexity and computation time. The operation of our proposed structure is as follows:

- The use of the Linear Array (ULA) to scan 180° to receive all elevations "θ"
- The use of the Circular Array (UCA) to perform a 360° scan to extract all azimuths "Φ " from received signals.

The organization of the manuscript is being as follows: The formalism of the Matrix Pencil based on uniform linear array model is presented in section 2. The MP method for the Uniform Circular Array is explained in section 3.The performance and the results of the 2-D MP method showed in section 4 and at the end the conclusion.

2. Matrix Pencil using ULA

In order to study our approach, we have started by the Matrix Pencil formalism by using the real Matrix to evaluate the direction

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of arrival of multiple impinged on the Uniform Linear Array [8]. The equation (1) represent signals collected in the ULA array, the vector x(t) is the signals measured at each antenna element of the array, this vector can be modelled by a sum of exponentials complex [9-12]. Therefore, n(t) is the noise. The observed voltage is given by:

$$\mathrm{y(t)} = x(t) + n(t) = \sum_{i=1}^{N} \mathrm{R_i} e^{\mathrm{s_i} t} + \mathrm{n(t)} \tag{1}$$

By using the equation one, the sum of signals are written as follow:

$$\mathrm{y(P)} = \sum_{\mathrm{i=1}}^{\mathrm{N}} \mathrm{R_i z_i}^{\mathrm{p}} + \mathrm{n(p)} \tag{2}$$

Where,

$$\mathrm{Zi} = \mathrm{e}^{\mathrm{j}\frac{2\pi}{\lambda}\mathrm{d}\sin(\theta)}, \text{ for i = 1, 2,…, N} \tag{3}$$

We assume that the factor damping αi = 0, in order to estimate the exact value of the angle θ. We construct the matrix Y to derive directly from the equation x(t), the column of this matrix Y is a windowed part of the original vector, {x (0) x (1) x (2) ... x (N) -1)}.

$$\mathrm{Y} = \begin{bmatrix} x(0) & x(1) & \cdots & x(L-1) \\ x(1) & x(2) & \cdots & x(L) \\ \vdots & \vdots & \ddots & \vdots \\ x(N-L) & x(N-L+1) & \cdots & x(N-L) \end{bmatrix}_{N-(L+1)\,x\,(L)} \tag{4}$$

L is the pencil parameter; it is chosen between N/3 and N/2 for noise filtering [13-14].The values of *L* are chosen in this range [15]. From the matrix *Y*, we define two sub-matrixes, say

$$\mathrm{Y_a} = \begin{bmatrix} x(0) & x(1) & \cdots & x(L-1) \\ x(1) & x(2) & \cdots & x(L) \\ \vdots & \vdots & \ddots & \vdots \\ x(N-L-1) & x(N-L) & \cdots & x(N-2) \end{bmatrix}_{(N-L)\,x\,(L)} \tag{5}$$

$$\mathrm{Y_b} = \begin{bmatrix} x(1) & x(1) & \cdots & x(L-1) \\ x(2) & x(2) & \cdots & x(L) \\ \vdots & \vdots & \ddots & \vdots \\ x(N-L) & x(N-L+1) & \cdots & x(N-1) \end{bmatrix}_{(N-L)\,x\,(L)} \tag{6}$$

We can also write

$$\mathrm{Y_a} = \mathrm{Z_a R\, Z_b} \tag{7}$$

$$\mathrm{Y_b} = \mathrm{Z_a\, R_0\, Z_0 Z_b} \tag{8}$$

$$\mathrm{Z_a} = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ Z_1 & Z_2 & \cdots & Z_M \\ \vdots & \vdots & \ddots & \vdots \\ {Z_1}^{(N-L-1)} & {Z_2}^{(N-L-1)} & \cdots & {Z_M}^{(N-L-1)} \end{bmatrix}_{(N-L)\,x\,(L)} \tag{9}$$

$$\mathrm{Z_b} = \begin{bmatrix} 1 & Z_1 & \cdots & {Z_1}^{(L-1)} \\ 1 & Z_2 & \cdots & {Z_2}^{(L-1)} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & Z_M & \cdots & {Z_M}^{(L-1)} \end{bmatrix}_{(M\,x\,L)} \tag{10}$$

$$\mathrm{Z_0} = \mathrm{diag}\,[\mathrm{Z_1, Z_2, \dots, Z_M}] \tag{11}$$

$$\mathrm{R_0} = \mathrm{diag}\,[\mathrm{R_1, R_2, \dots, R_M}] \tag{12}$$

Now, we consider the Matrix Pencil

$$\mathrm{Y_b} - \lambda \mathrm{Y_a} = \mathrm{Z_a} - \mathrm{R_0}\,[\mathrm{Z_0} - \lambda \mathrm{I}]\mathrm{Z_b} \tag{13}$$

We use the identity matrix I of dimension (MxM), which can give us an idea about the rank of Yb - λYa. This rank will be M, if only if M≤L≤N-M [15-16]. While, if λ = Zi, i = 1, 2, ..., M the ith line of [Z0 -λI] = 0, the rank of this matrix is then (Mx1) . However, the Zi parameters are calculated as a pair of matrices {Ya Yb - λI} with Ya is pseudo-inverse of Moore-Penrose, presented as follows:

$${\mathrm{Y_a}}^{+} = \left\{{\mathrm{Y_a}}^{\mathrm{H}}\,\mathrm{Y_a}\right\}^{-1}{\mathrm{Y_a}}^{\mathrm{H}} \tag{14}$$

The DOA is obtained from :

$$\theta_{\mathrm{i}} = \sin^{-1}\left(\frac{\mathrm{Im}\,(\log \mathrm{Z_i})}{\pi \mathrm{d}}\right) \tag{15}$$

Where Zi is defined in eq. (3)

3. Matrix Pencil using UCA

In this section, we use a UCA of separate antennas distributed on the axes ox and oy, with dx = dy = d. The array receives the signals with incidence angles of (θq, Φq), which are respectively represent the elevation and azimuth. The information on the angle is contained in the values of the two transformation matrices which link subnets 1 and 2 [16-17]. The expressions αx and αy are in the following form:

$$\alpha_{xi} = \exp(\,\mathrm{j}\left(\frac{2\pi\,\Delta}{\lambda_0}\right)\sin\theta\mathrm{i}\cos\Phi\,\mathrm{i}\,) \tag{16}$$

$$\alpha_{\mathrm{xi}} = \exp(\,\mathrm{j}\left(\frac{2\pi\,\Delta}{\lambda_0}\right)\sin\theta\mathrm{i}\sin\Phi\mathrm{i}) \tag{17}$$

Expressions of the elevation and azimuth are giving by the equations:

$$\theta\mathrm{i} = \mathrm{Arcsin}\,[\left(\frac{-\mathrm{j}\lambda_0}{2\pi\Delta}\right)\sqrt{(\mathrm{Ln}\,\alpha_{\mathrm{xi}})^2 + \left(\mathrm{Ln}\,\alpha_{\mathrm{yi}}\right)^2}\,] \tag{18}$$

$$\Phi\,\mathrm{i} = \mathrm{Arctg}\left(\frac{\mathrm{Ln}\alpha_{\mathrm{xi}}}{\mathrm{Ln}\,\alpha_{\mathrm{yi}}}\right) \tag{19}$$

With i= 1, 2, ... , Ms.

4. Results and discussion

We present in this section the simulations results of the Matrix Pencil for the proposed structure by using the Matrix Pencil. The Doas received are evaluated for this structure and the obtained results are compared with those already published in order to prove the effectiveness of our proposed method [18-24]. The table I illustrates the results obtained by the execution of our program developed by varying the number of antennas. We opted for the RMSE criterion to assess accuracy.

So, we have studied the antenna structure of 100 and 95 elements. We assumed that five antennas no longer worked. Then we tested our program to minimize the error of these five antennas while varying the number of samples from 1 to 4 to deal with this failure when implementing our method.

Table 1. Proposed Method for 100 and 95 antennas

Number of antennas		θ°_{in}	θ°_{out}	*RMSE*
100	**[18]**	0	0.0004	0.0019
		5	5.0001	
		10	9.9948	
		15	15.0014	
		20	20.0045	
		30	29.9984	
	Proposed Method	0	0.0000	0.0006
		5	5.0001	
		10	9.9991	
		15	15.0001	
		20	20.0000	
		30	30.0001	
95	**[18]**	0	0.0067	0.0023
		5	5.0024	
		10	10.0014	
		15	14.9976	
		20	19.9965	
		30	29.9991	
	Proposed Method	0	0.0005	0.0018
		5	5.0006	
		10	10.0008	
		15	15.0009	
		20	20.0000	
		30	29.9999	

To deepen our work, we studied the variation of the angle θ deviation by varying the number of antennas from 7 to 14. The obtained results are presented in table II.

Table 2. Results of proposed method with various antennas

Number of antennas		θ°_{in}	θ°_{pencil}	$\Delta\theta^{\circ}_{pencil}$
7	**[19]**	30	37.5900	0.253
		60	61.8400	0.030
	Proposed Method	30	30.0017	0.226
		60	60.0928	0.025
8	**[19]**	30	30.1800	0.006
		60	61.6400	0.027
	Proposed Method	30	30.0477	0.003
		60	60.0267	0.018
10	**[19]**	30	30.3100	0.010
		0.2	0.2100	0.05
	Proposed Method	30	30.0004	0.006
		0.2	0.2010	0.005
14	**[19]**	30	30.1700	0.005
		60	59.4900	0.0085
	Proposed Method	30	29.9975	0.0026
		60	60.0009	-0.0033

We conclude that even in the presence of this failure; Matrix Pencil assures that estimation of the directions of arrivals remains stable even if the conditions change with a max of error of equal to 0.030% and a min of error equal to 0.017%.

The results presented in table. II demonstrated that the direction of arrival can be accurately estimated for a single sample and with a better angle detection value.

As shown in figure 1; we used a unequal signals power for azimuth and elevation (133.6°,137.8°) and (78.6°, 82.4°), respectively. One power value is 7 dBm and the other is 5 dBm. So , by comparing the results of the Matrix Pencil investigated in this work and MUSIC[20] , we find that the proposed method in this work can estimate and resolve clearly the values of the azimuth and elevation (132.4°, 136.2°) and (78°, 84°), and the peaks are sharp compared to [21].

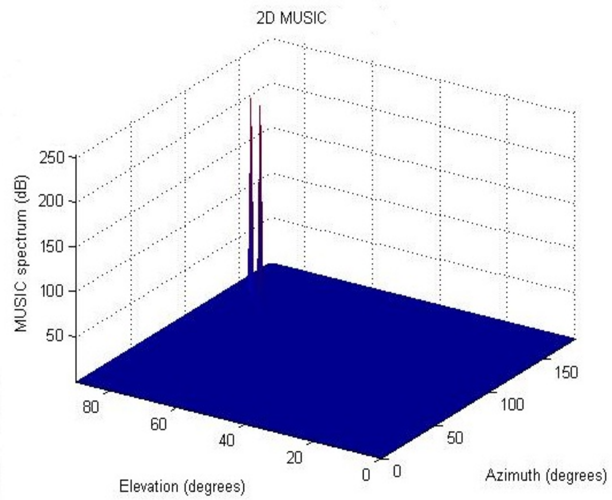

Figure 1. Elevation and azimuth for (133.6°, 137.8°) and (78.6°, 82.4°)

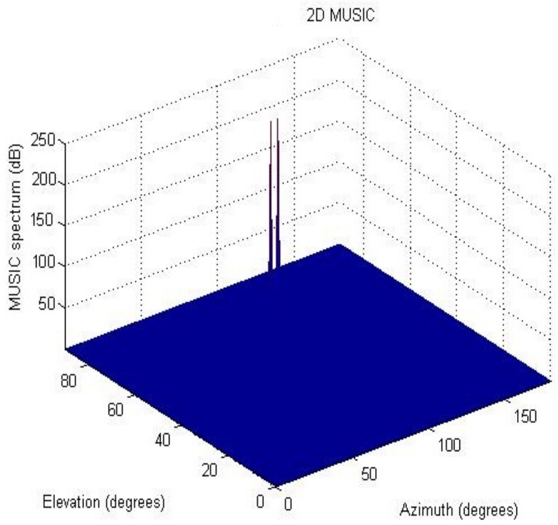

Figure 2. Elevation and azimuth for (128.4°, 116°) and (78°, 84°)

In the second simulation , we have changed the azimuth and elevation values ,(128.4°,116°) and (78°,84°), and the power signals is 7 dBm and 5 dBm. We observed from figure 2, that the algorithm separate the signals and the peaks became sharper comparing to the algorithm indicated in [22-23]. Furthermore, the direction of arrival estimations for the angles (128.4°,116°) and (78°,84°), is more accurate than MUSIC method .

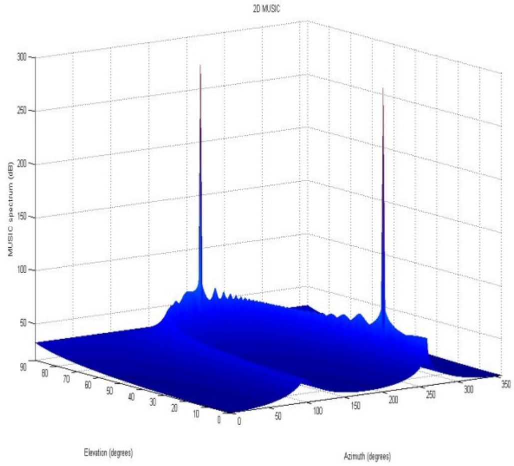

Figure 3. DOA for 5.24 GHZ

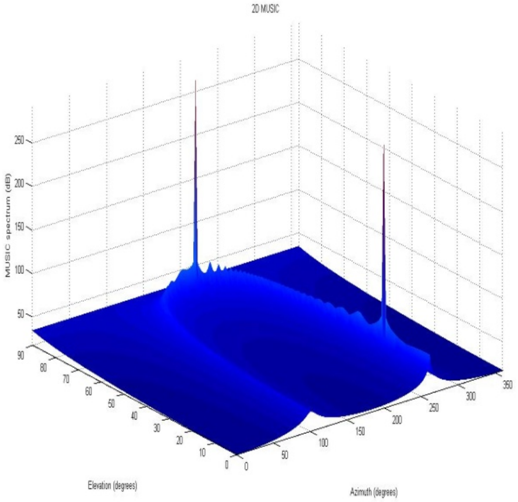

Figure 4. DOA for 7.26 GHz

From figures.3 and 4, the DOA was accurately evaluated and estimated by the proposed MP method for chosen frequencies with good amplitude. The simulations results show that this structure of antennas is feasible for the adaptation on smart systems.

Table 3. Method MUSIC and Pencil for different SNR

Reference	SNR	UCA angles (deg°)		UCA errors (deg°)	
		θ	***Φ***	***θ***	***Φ***
[24]	**3**	-37.799999	57.600002	0.200001	0.600002
Proposed method	**3**	-37.600000	57.100002	0.400000	0.100002
[24]	**-3**	-37.400000	56.500000	0.600000	-0.500000
Proposed method	**-3**	-37.399998	58.800003	0.600002	1.800003

In the following step, the proposed MP method is compared with MUSIC method indicated at [24], under the same conditions shown in table III with two angles -38° and -57°. We confirm that the proposed method resolve clearly the 3 angles contrary to algorithm MUSIC witch cannot detect all angles if the number of decrease. Our results give less error margin to estimate DOA. Table III estimates the margin error for the estimation of the angles θ and Φ for different values of the SNR.

Analysing table III, we conclude that:

- For a SNR of -3 dB, the Matrix Pencil has a better estimation than the MUSIC, with an error rate of 0.75% for the circular network and 0.96% for the linear network.
- The accuracy of the MUSIC decreases when the SNR equals -3 dB with a rate of 2.1% for the linear network and 1.27% for the circular network.
- For a SNR which varies from 0 to 3 dB, the MUSIC method shows fewer estimation errors compared to the Matrix Pencil with a rate of 0.35%.

- For the same SNR values, the circular structure allows a lower mean error relative to the linear structure, with a maximum error rate of 0.42%.
- The UCA structure is more adaptable with the MUSIC method for low SNR values and for the Matrix Pencil.

- Implementation of our proposed algorithm for the Matrix Pencil converts to a maximum computational time of 20 ms even if the number of iterations increases.

In this research, we have used the Matrix Pencil for a divided array. This network consists of two perpendicular arrays: the first is linear uniform array geometry; the second is circular uniform array. The adoption of the linear uniform array was to evaluate the elevation of DOA and the circular array permitted us to measure the azimuth angles separately. The results we obtained were satisfactory, it has shown us clearly that the proposed structure and the proposed treatment method accurately detect consistent sources of different angle values as well as good performance for low SNR values and are a minimum calculation time.

The authors declare no conflict of interest.

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