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Performance Analysis and Enhancement of Spline Adaptive Filtering based on Adaptive Step-size Variable Leaky Least Mean Square Algorithm

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ABSTRACT

This paper presents an adaptive step-size and variable leaky least mean square algorithm based on nonlinear adaptive filter with the adaptive lookup table using spline interpolation. An adaptive step-size approach is proposed with the energy of squared previous and present errors to boost up the convergence rate. A modified variable leaky mechanism is proposed with the optimal leaky parameter by using the recursion form. The proposed algorithm merges an adaptive step-size and a modified variable leaky method with least mean square algorithm for linear and nonlinear network part of spline adaptive filtering in term of fast convergence enhancement. Experimental results demonstrate that proposed algorithm can notably achieve a competitive performance on the convergence rate in comparison with the conventional least mean square algorithm for spline adaptive filter. Simulation results suggest that mean square error performance of proposed algorithm can be partially assessed using adaptive step-size with the variable leaky parameters indicating better than the conventional least mean square algorithm by 16.76%.

1 Introduction

This paper is an extension of work originally presented in the International Electrical Engineering Congress [1], which has been proposed the variable leaky mechanism based on the least mean square (LMS) algorithm for spline adaptive filter. The combination of a linear finite impulse response (FIR) filter and a nonlinear adaptive lookup table (LUT) based on the spline interpolation is called the spline adaptive filtering (SAF) with the adaptation process [2]. It is a class of nonlinear adaptive filtering with a spline function [1]-[12].

According to the practical models, the linear adaptive filter should be inadequate [3]. The models of nonlinear dynamic systems have demonstrated with more robust performance. Linear adaptive filtering should be insufficient in real-world models. Moreover, many dynamic systems using model of nonlinear structure have been extended to the operating model.

Based on the SAF, many research works have achieved efficiently in the practical system identification. In [2], the authors have presented the sign approach with the normalized version of least mean square (NLMS) based on Wiener spline adaptive filter in order to fight against the impulsive noise environment. In [4], the authors have analyzed the convergence and stability analysis of SAF based on LMS algorithm. A steady-state performance of SAF has been examined in [5].

In order to model the nonlinear system identification, the SAF is more attention for the practical use [6],[7]. The linear time-invariant model with the cubic spline function [8] can obtain the good performance working with the adaptive lookup table (LUT) [9] and the spline basis matrix on the adaptive control points coefficient vector.

Applications of SAF architecture have been applied in the infinite impulse response [10] and the system environment with impulsive noise [11]. A set-membership mechanism with the Wiener spline adaptive filtering and normalized least M-estimate algorithm have been obtained significantly the fast convergence in the environment of impulsive noise [11]. In [12], the authors have proposed the Hammerstein function with SAF based on LMS algorithm for improving the convergence rate. Further, a SAF based on the maximum correntropy criterion [13], [14] has been discussed that correntropy is robustness to non-Gaussian noise.

Least mean square (LMS) algorithm which is simple and low computation has been widely used [15]. Most researchers have modified accurately on LMS and NLMS algorithms [16] and [17], the variable step-size LMS [18], the sign mechanism with NLMS [2] and so on.

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As stated in a steady-state of convergence analysis, a variable leaky mechanism has been depicted in use of tracking the systems. In [19], a variable leaky LMS algorithm based on the greedy heuristic algorithm has been leveraged to enhance with the high eigenvalue input. In [20], the authors have implemented the variable leaky LMS in the field programmable gate array implementation. The results have been shown that the algorithm can explore the various digital filter architecture.

Based on the variable leaky LMS algorithm which has been orchestrated against and attenuating the drifting [21]. In the field of adaptive signal processing, a variable leaky based on the orthogonal gradient adaptive algorithm with the minimized cost function has been applied for the wireless communications [22].

The novelty of this paper is to merge an adaptive step-size and a modified variable leaky method with least mean square algorithm for linear and nonlinear network part of spline adaptive filtering in term of fast convergence enhancement. The analysis of the convergence performance of the proposed algorithm is derived in forms of the properties and mean square error performance.

In this paper, we organize this paper as follows. Section 2 explains shortly in the structure of SAF. Next, Section 3 introduces the modified adaptive step-size mechanism and the modified variable leaky criterion on the minimization cost function based on the LMS algorithm. Section 4 describes the performance analysis of the proposed adaptive step-size and variable leaky LMS algorithm in terms of the properties and the mean square error performance. Further, the experiment results and discussion show in Section 5 and Section 6, respectively. Finally, Section 7 summarizes the proposed algorithm.

2 Spline Adaptive Filter

Spline adaptive filter (SAF) is namely a combination of linear and nonlinear structures shown in Fig. 1, where $\mathbf{x}_k$ is the input of SAF structure and y_k is the output of system.

The objective is that adaptive lookup table in the nonlinear structure generates an output of SAF y_k nearly to a desired sequence d_k as

$$d_k = y_k + e_k\ , \tag{1}$$

where the error of system e_k should be small. Thus, the adaptive FIR filter brings an output ϕ_k in the linear structure, while the input signal is a sequence of x_k at the linear structure.

An adaptive FIR filter output ϕ_k is given by [3]

$$\phi_k = \mathbf{w}_k^T\,\mathbf{x}_k\ , \tag{2}$$

where $\mathbf{w}_k$ is the update FIR coefficient vector.

Following [3], the SAF output y_k can be examined by following

$$y_k = \mathbf{u}_k^T\,\mathbf{C}_b\,\mathbf{g}_{m,k}\ , \tag{3}$$
$$\mathbf{v}_k = [v_k^3,\ v_k^2,\ v_k,\ 1]^T\ , \tag{4}$$

where $\mathbf{g}_{m_k}$ is the coefficient of control point vector and $\mathbf{C}_b$ is a spline basis matrix.

The local parameter v_k and index m are usually indicated as [5]

$$v_k = \frac{\phi_k}{\Delta x} - \left\lfloor \frac{\phi_k}{\Delta x} \right\rfloor\ , \tag{5}$$
$$m = \left\lfloor \frac{\phi_k}{\Delta x} \right\rfloor + \frac{P-1}{2}\ , \tag{6}$$

where Δx is the uniform space between two adjacent-coefficients of control points [6], P is the size of control point coefficient, and the floor operator $\lfloor\cdot\rfloor$ is applied.

By using the minimized objective function based on least mean square (LMS) [5], it becomes

$$J_{\mathbf{w},\mathbf{g}}^{\mathrm{LMS}}(k) = \min_{\mathbf{w},\mathbf{g}} \left\{ \frac{1}{2} \mid e_k \mid^2 \right\}, \tag{7}$$

where *a priori* error e_k is defined as

$$e_k = d_k - y_k = d_k - \mathbf{v}_k^T\,\mathbf{C}_b\,\mathbf{g}_{m,k}\ . \tag{8}$$

The update FIR coefficient vector $\mathbf{w}_k$ can compute by the gradient vector of (7) with respect to the coefficients $\mathbf{w}_k$ in terms of the recursion form as [3]

$$\mathbf{w}_{k+1} = \mathbf{w}_k - \mu_w\,\nabla J_{\mathbf{w}}^{\mathrm{LMS}}\ , \tag{9}$$

where μ_w is a step-size and $\nabla J_{\mathbf{w}}^{\mathrm{LMS}}(k)$ is the gradient vector for $\mathbf{w}_k$.

Similarly, the update control points vector $\mathbf{g}_{m,k}$ can be expressed by the gradient vector of (7) with respect to the coefficients $\mathbf{g}_{m,k}$ [3]

$$\mathbf{g}_{m,k+1} = \mathbf{g}_{m,k} - \mu_g\,\nabla J_{\mathbf{g}}^{\mathrm{LMS}}(k)\ , \tag{10}$$

where μ_g is a step-size and $\nabla J_{\mathbf{g}}^{\mathrm{LMS}}(k)$ is the gradient vector for $\mathbf{g}_{m,k}$. Thus, the FIR weight vector $\mathbf{w}_k$ and the control points update coefficient vector $\mathbf{g}_{m,k}$ are the particularly simple forms as [5]

$$\mathbf{w}_{k+1} = \mathbf{w}_k + \mu_w\,\mathbf{v}_k'\,\mathbf{C}_b\,\mathbf{g}_{m,k}^T\,\mathbf{x}_k\,e_k\ , \tag{11}$$
$$\mathbf{g}_{m,k+1} = \mathbf{g}_{m,k} + \mu_g\,\mathbf{C}_b^T\,\mathbf{v}_k\,e_k\ , \tag{12}$$

where $\mathbf{v}_k'$ is given by

$$\mathbf{v}_k' = [3\,v_k^2,\ 2\,v_k,\ 1,\ 0]^T\ . \tag{13}$$

and the local parameter $\mathbf{v}_k$ is given in (5).

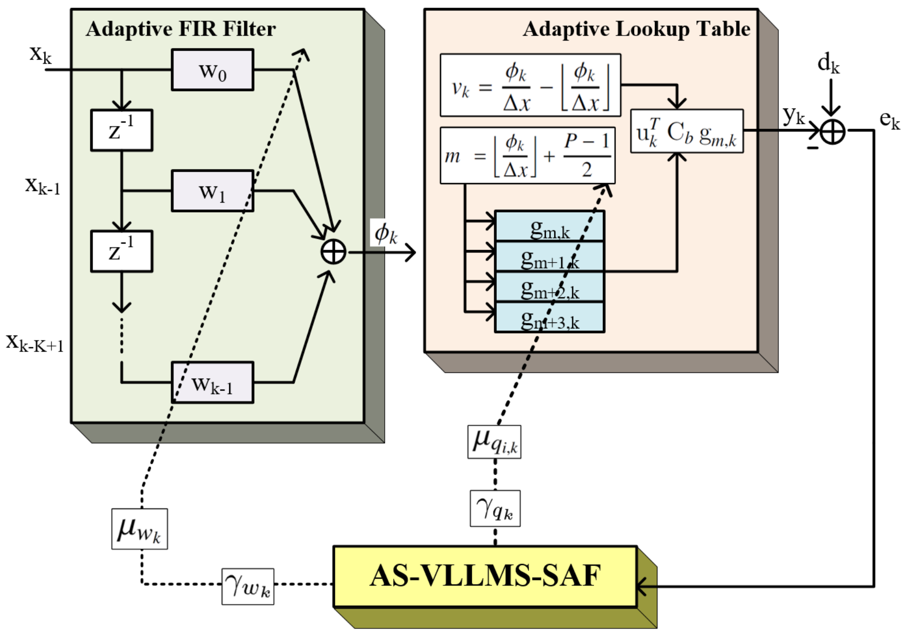

Figure 1: Linear-Nonlinear structure of proposed adaptive step-size variable leaky LMS algorithm for spline adaptive filter (AS-VLLMS-SAF).

3 Proposed Adaptive Step-size and Variable Leaky Least Mean Square based on Spline Adaptive Filter

Based on the least mean square (LMS) algorithm, the advantage of the leaky LMS algorithm is that it avoids the drift of weights [23]. Meanwhile, the step-size parameter is an efficient approach to improve the convergence rate [16].

Following [1] and [11], the cost function using adaptive step-size and variable leaky criterion for SAF based on least mean square (AS-VLLMS-SAF) algorithm can be minimized as

$$J(\mathbf{w}_k, \mathbf{g}_{m,k}) = \min_{\mathbf{w}_k, \mathbf{g}_{m,k}} \left\{ \frac{1}{2} \mid e_k \mid^2 + \gamma_w \|\mathbf{w}_k\|^2 + \gamma_g \|\mathbf{g}_k\|^2 \right\}, \tag{14}$$

where γ_w and γ_g are the leaky paramters for the linear FIR coefficient vector $\mathbf{w}_k$ and the control points weight vector $\mathbf{g}_{m,k}$, respectively. The *a priori* error of system e_k is given as

$$e_k = d_k - \mathbf{v}_k^T \mathbf{C}_b \mathbf{g}_{m,k} . \tag{15}$$

Considering the chain rule on the cost function in (14) with respect to (w.r.t) $\mathbf{w}_k$, we get

$$\begin{aligned} \frac{\partial J(\mathbf{w}_k, \mathbf{g}_{m,k})}{\partial \mathbf{w}_k} &= \left\{ -e_k \frac{\partial y_k}{\partial \mathbf{v}_k} \frac{\partial \mathbf{v}_k}{\partial \phi_k} \frac{\partial \phi_k}{\partial \mathbf{w}_k} + \gamma_w \mathbf{w}_k \right\} \\ &= \left\{ -\frac{e_k}{\Delta x} \mathbf{v}_k' \mathbf{C} \mathbf{g}_{m,k} \mathbf{x}_k + \gamma_w \mathbf{w}_k \right\}, \end{aligned} \tag{16}$$

where $\mathbf{v}_k'$ is defined as

$$\mathbf{v}_k' = [3v_k^2 \;\; 2v_k \;\; 1 \;\; 0] . \tag{17}$$

The gradient of cost function in (14) w.r.t $\mathbf{g}_{m,k}$ can be expressed by using the chain rule in the form of vector below

$$\begin{aligned} \frac{\partial J(\mathbf{w}_k, \mathbf{g}_{m,k})}{\partial \mathbf{g}_{m,k}} &= \left\{ -e_k \frac{\partial y_k}{\partial \mathbf{v}_k} \frac{\partial \mathbf{v}_k}{\partial \phi_k} \frac{\partial \phi_k}{\partial \mathbf{g}_{m,k}} + \gamma_g \mathbf{g}_{m,k} \right\} \\ &= \{ -e_k \mathbf{C}^T \mathbf{v}_k + \gamma_g \mathbf{g}_{m,k} \} . \end{aligned} \tag{18}$$

Hence, the proposed update linear FIR coefficient vector $\mathbf{w}_k$ of AS-VLLMS-SAF algorithm is the stochastic adaptation formula as

$$\mathbf{w}_{k+1} = \mathbf{w}_k - \mu_{w_k} \frac{\partial J(\mathbf{w}_k, \mathbf{g}_{m,k})}{\partial \mathbf{w}_k} , \tag{19}$$

where μ_{w_k} is a adaptive step-size parameter at symbol k.

By substituting (16) into (19), the proposed update tap-weight vector $\mathbf{w}_k$ can be expressed as

$$\therefore \mathbf{w}_{k+1} = (\mathbf{1} - \mu_{w_k}\gamma_{w_k}) \mathbf{w}_k + \frac{\mu_{w_k}}{\Delta x} \mathbf{v}_k' \mathbf{C}_b \mathbf{g}_{m,k} \mathbf{x}_k e_k , \tag{20}$$

where γ_{w_k} is a variable leaky parameter for $\mathbf{w}_k$. It is noticed that $(1 - \mu_{w_k}\gamma_{w_k})$ is also defined as the leakage factor [23] for updated weight $\mathbf{w}_k$ in which its value is generally close to 1.

The adaptive control points coefficient vector $\mathbf{g}_{m,k}$ of AS-VLLMS-SAF becomes

$$\mathbf{g}_{m,k+1} = \mathbf{g}_{m,k} - \mu_{\mathbf{g}_k} \frac{\partial J(\mathbf{w}_k, \mathbf{g}_{m,k})}{\partial \mathbf{g}_{m,k}} , \tag{21}$$

By substituting (18) into (21), the proposed control points vector $\mathbf{g}_{m,k}$ is given as

$$\therefore \mathbf{g}_{m,k+1} = (\mathbf{1} - \mu_g \gamma_{g_k})\, \mathbf{g}_{m,k} + \mu_q\, \mathbf{C}_b^T\, \mathbf{v}_k\, e_k\ , \tag{22}$$

where μ_g is a step-size parameter and γ_{g_k} is a variable leaky parameter for $\mathbf{g}_{m,k+1}$. It is noted that $(1 - \mu_g \gamma_{g_k})$ is also defined as the leakage factor [23] for updated weight $\mathbf{g}_{m,k+1}$ in which its value is generally close to 1.

3.1 Modified Variable Leaky mechanism

Following [24], we introduce the modified variable leaky algorithm with the optimal leaky parameter for weights $\mathbf{w}_k$ and $\mathbf{g}_{m,k}$ in the recursion form as

$$\gamma_{w_k} = \gamma_{w_{k-1}} + \rho_w\, \gamma_w^{\text{opt}}\ , \tag{23}$$

$$\gamma_{g_k} = \gamma_{g_{k-1}} + \rho_g\, \gamma_g^{\text{opt}}\ , \tag{24}$$

where ρ_w, ρ_g and γ_w^{opt}, γ_g^{opt} are the adaptation rate and the optimal leaky parameters of weights $\mathbf{w}_k$ and $\mathbf{g}_{m,k}$.

We rewrite (20) as

$$\mathbf{w}_{k+1} = \mathbf{w}_k - \mu_{w_k} \gamma_{w_k} \mathbf{w}_k + \frac{\mu_{w_k}}{\Delta x}\, \mathbf{v}'_k\, \mathbf{C}_b\, \mathbf{g}_{m,k}\, \mathbf{x}_k\, e_k\ , \tag{25}$$

Assumption 1: *We consider the steady-state value of $E\{\mathbf{w}_{k+1}\}$ for $k \to \infty$ by*

$$E\{\mathbf{w}_{k+1}\} \simeq E\{\mathbf{w}_k\}.$$

We determine the optimal leaky parameter for $\mathbf{w}_k$ by using this assumption above in (25), we arrive at

$$\gamma_w^{\text{opt}} \simeq \frac{\mathbf{v}'_k\, \mathbf{C}_b^T\, \mathbf{g}_{m,k}\, \mathbf{w}_k^{-1}\, \mathbf{x}_k\, e_k}{\Delta x}\ . \tag{26}$$

Therefore, the modified variable leaky algorithm in the recursion form can be expressed as

$$\gamma_{w_k} = \gamma_{w_{k-1}} - \rho_w\, \frac{\mathbf{v}'_k\, \mathbf{C}_b^T\, \mathbf{g}_{m,k}\, \mathbf{w}_k^{-1}\, \mathbf{x}_k\, e_k}{\Delta x}\ , \tag{27}$$

where ρ_w is an adaptation rate for $\mathbf{w}_k$.

Consequently, the *a posterior* AS-VLLMS error [1] is rewritten as

$$e_{p_k}^{\text{VL-LMS}} = (1 - \mu_{g_k}\, \mathbf{v}_k\, \mathbf{C}_b^T\, \mathbf{v}_k\, \mathbf{C}_b)\, e_k + \mu_{g_k}\, \gamma_{g_k}\, \mathbf{v}_k\, \mathbf{C}_b^T\, \mathbf{g}_{m,k}\ . \tag{28}$$

Assumption 2: *We consider the steady-state value of proposed algorithm is stable for $k \to \infty$ by*

$$E\{e_{p_k}^{\text{VL-LMS}}\} \simeq E\{e_k\}.$$

By using Assumption 2 in (28), the optimal leaky parameter can be defined as

$$\gamma_g^{\text{opt}} \simeq \frac{\Omega_k^T\, \Omega_k\, e_k}{\Omega_k^T\, \mathbf{g}_{m,k}}\ , \tag{29}$$

where $\Omega_k = \mathbf{v}_k^T\, \mathbf{C}$.

Table 1: Proposed Variable Leaky Least Mean Square based on Spline Adaptive Filtering with the adaptive step-size (AS-VLLMS-SAF) algorithm.

Initialize : $\mathbf{w}(0) = \delta\, \mathbf{I}$ and $\mathbf{I}$ is the identity matrix.

$$\mathbf{w}_k = [\ w_0\ w_1\ \ldots\ w_{K-1}\ ]$$
$$\mathbf{x}_k = [\ x_k\ x_{k-1}\ \ldots\ x_{k-K+1}\ ]$$
$$\mathbf{g}_{m,k} = [\ g_{m,k}\ \ g_{m+1,k}\ \ g_{m+2,k}\ \ g_{i+3,k}\ ]$$

for $k = 0, 1, 2, \ldots, K-1$.

$$\begin{aligned}
\phi_k &= \mathbf{w}_k^T\, \mathbf{x}_k \\
v_k &= \frac{\phi_k}{\Delta x} - \left\lfloor \frac{\phi_k}{\Delta x} \right\rfloor \\
m &= \left\lfloor \frac{\phi_k}{\Delta x} \right\rfloor + \frac{P-1}{2} \\
\mathbf{v}_k &= [v_k^3,\ v_k^2,\ v_k,\ 1]^T \\
\mathbf{v}'_k &= [3v_k^2,\ 2v_k,\ 1,\ 0]^T \\
e_k &= d_k - \mathbf{v}_k^T\, \mathbf{C}_b\, \mathbf{g}_{m,k} \\
\gamma_{w_k} &= \gamma_{w_{k-1}} + \rho_w\, \frac{\mathbf{v}'_k\, \mathbf{C}_b^T\, \mathbf{g}_{m,k}\, \mathbf{w}_k^{-1}\, \mathbf{x}_k\, e_k}{\Delta x} \\
\gamma_{q_k} &= \gamma_{q_{k-1}} + \rho_q\, \frac{\Omega_k^T\, \Omega_k\, e_k}{\Omega_k^T\, \mathbf{g}_{m,k} + \epsilon} \\
\Omega_k &= \mathbf{v}_k^T\, \mathbf{C}_b \\
\mu_{w_k} &= \alpha_w\, \mu_{w_{k-1}} + \beta_w\, \xi_k^2 \\
\xi_k &= \lambda\, \xi_{k-1} + (1-\lambda)\, e_{k-1}^*\, e_k \\
\mu_{q_k} &= \alpha_q\, \mu_{q_{k-1}} + \beta_q\, e_{k-1}^*\, e_k \\
\mathbf{w}_{k+1} &= \mathbf{w}_k - \mu_{w_k} \gamma_{w_k} \mathbf{w}_k + \frac{\mu_{w_k}}{\Delta x}\, \mathbf{v}'_k\, \mathbf{C}_b^T\, \mathbf{g}_{m,k}\, \mathbf{x}_k\, e_k \\
\mathbf{q}_{i,k+1} &= \mathbf{g}_{m,k} - \mu_{q_k} \gamma_{q_k} \mathbf{g}_{m,k} + \mu_{q_{m,k}}\, \mathbf{C}_b^T\, \mathbf{v}_k\, e_k
\end{aligned}$$

end

Therefore, the modified variable leaky algorithm can be expressed in the recursion method as

$$\gamma_{g_k} = \gamma_{g_{k-1}} - \rho_q\, \frac{\Omega_k^T\, \Omega_k\, e_k}{\Omega_k^T\, \mathbf{g}_{m,k} + \epsilon}\ , \tag{30}$$

where ρ_g is an adaptation rate for $\mathbf{g}_{m,k}$ and ϵ is a regularization parameter with a small constant.

3.2 Modified Adaptive Step-size Approach

According to the better convergence of proposed AS-VLLMS-SAF algorithm, the step-size parameter should be adapted recursively.

The behavior of convergence rate is which the algorithm starts converging, the value of step-size parameter should be large in order to boost up the learning rate of convergence. At closely to the steady-state, the step-size parameter might be decreased adequately to get the lower adjustment. This leads to adjust accordingly the step-size parameter.

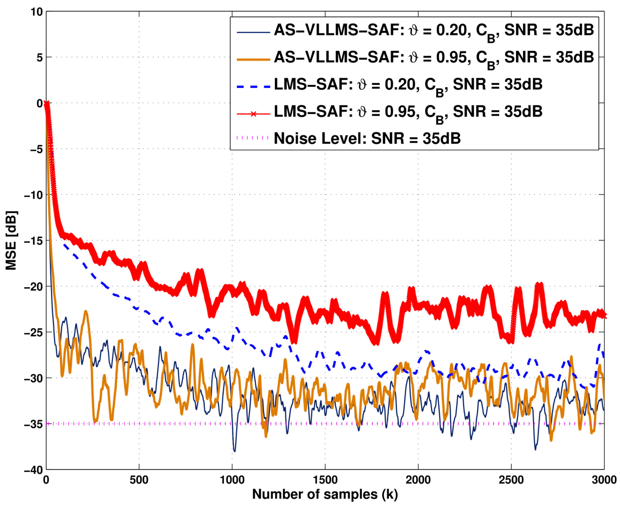

Figure 2: MSE of proposed AS-VLLMS-SAF and LMS-SAF [3] on the *B-spline* matrix $\mathbf{C}_B$ with the different $\vartheta = 0.20, 0.95$ and SNR = 35dB.

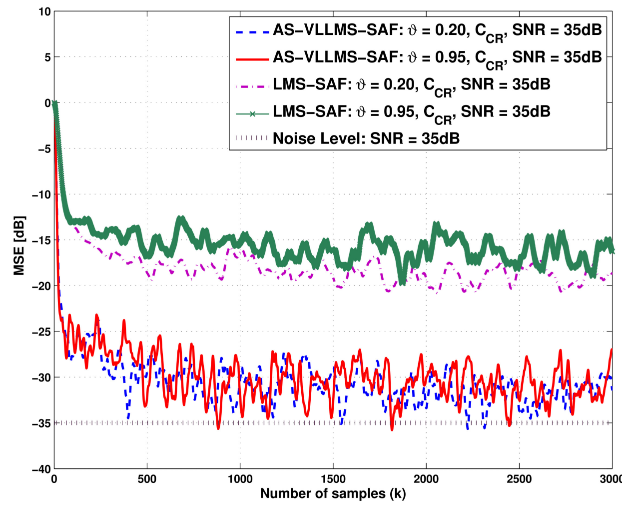

Figure 3: MSE of proposed AS-VLLMS-SAF and LMS-SAF [3] on the *Catmul-Rom* matrix $\mathbf{C}_{CR}$ with $\vartheta = 0.20, 0.95$ and SNR=35dB.

We assume that the step-size parameters μ_{w_k} and μ_{g_k} are regulated by squared estimated error. Hence, the proposed adaptive step-size μ_{w_k} for weight $\mathbf{w}_k$ based on AS-VLLMS algorithm is introduced for spline adaptive filtering as [4]

$$\mu_{w_k} = \alpha_w \, \mu_{w_{k-1}} + \beta_w \, \xi_k^2 \,, \tag{31}$$

$$\xi_k = \lambda \, \xi_{k-1} + (1-\lambda) \, e_{k-1}^* \, e_k \,, \tag{32}$$

and the proposed adaptive step-size μ_{g_k} for weight $\mathbf{g}_{m,k}$ can be expressed as

$$\mu_{g_k} = \alpha_g \, \mu_{g_{k-1}} + \beta_g \, e_{k-1}^* \, e_k \,, \tag{33}$$

A summary of proposed variable leaky least mean square for spline adaptive filtering based on adaptive step-size algorithm (AS-VLLMS) is depicted in Table 1.

4 Performance Analysis

In this section, we analyze the convergence performance of the proposed variable leaky LMS based on SAF. Compared to the LMS-SAF algorithm, the proposed AS-VLLMS algorithm is presented by introducing an additional concept.

Variable leaky algorithm is used in the iterative of weights $\mathbf{w}_k$ and $\mathbf{g}_{m,k}$, respectively. So, the computational complexity of AS-VLLMS-SAF is more complex than that of the LMS-SAF algorithm.

4.1 Properties of AS-VLLMS Algorithm

The properties of adaptive step-size and variable leaky LMS algorithm may be derived by examining the behaviour of $E\{\mathbf{w}_k\}$ and $E\{\mathbf{g}_{m,k}\}$ of SAF.

Taking expectation value and using the independence assumption [25], we then have

$$E\{\mathbf{w}_{k+1}\} = (1 - \mu_{w_k}\gamma_{w_k}) \, E\{\mathbf{w}_k\} + \frac{\mu_w}{\Delta x} \, E\{\mathbf{v}_k' \, \mathbf{C}_b \, \mathbf{g}_{m,k} \, \mathbf{x}_k \, e_k\} \,. \tag{34}$$

Specially, we note that if $\mathbf{w}_k$ converges to the steady-state solution, we get

$$\lim_{k\to\infty} E\{\mathbf{w}_k\} = \frac{\mathbf{v}_k' \, \mathbf{C}_b \, \mathbf{g}_{m,k}^T \, \mathbf{x}_k \, e_k}{\gamma_{g_k} \Delta x} \,. \tag{35}$$

In a similar fashion, we substitute *a priori* error e_k in (15) into (22), then we have

$$\mathbf{g}_{m,k+1} = \left[\mathbf{I} - \mu_{g_k} \{\gamma_{g_k} \, \mathbf{I} + \mho_k^T \, \mho_k\}\right] \mathbf{g}_{m,k} + \mu_{g_k} \, \mho_k \, d_k \,, \tag{36}$$

where $\mho_k$ is given by

$$\mho_k = \mathbf{C}_b^T \, \mathbf{v}_k \,. \tag{37}$$

Thus, we take the expectation value and using the independence assumption [25], it becomes

$$E\{\mathbf{g}_{m,k+1}\} = \left[\mathbf{I} - \mu_{g_k}\left\{\mathbb{R}_\mho + \gamma_{g_k} \, \mathbf{I}\right\}\right] E\{\mathbf{g}_{m,k}\} + \mu_{g_k} \, \mathbf{r}_{d\mho} \,, \tag{38}$$

where $\mathbb{R}_\mho$ and $\mathbf{r}_{d\mho}$ are defined by

$$\mathbb{R}_\mho = \mho_k^T \, \mho_k \,, \tag{39}$$

$$\mathbf{r}_{d\mho} = d_k \, \mho_k \,. \tag{40}$$

Clearly, we note that if $\mathbf{g}_{m,k}$ converges to the steady-state solution, we have

$$\lim_{k\to\infty} E\{\mathbf{g}_{m,k}\} = \left(\mathbb{R}_\mho + \gamma_{g_k} \, \mathbf{I}\right)^{-1} \mathbf{r}_{d\mho} \,. \tag{41}$$

It is noticed that the leakage coefficient introduces a bias into the steady-state solution.

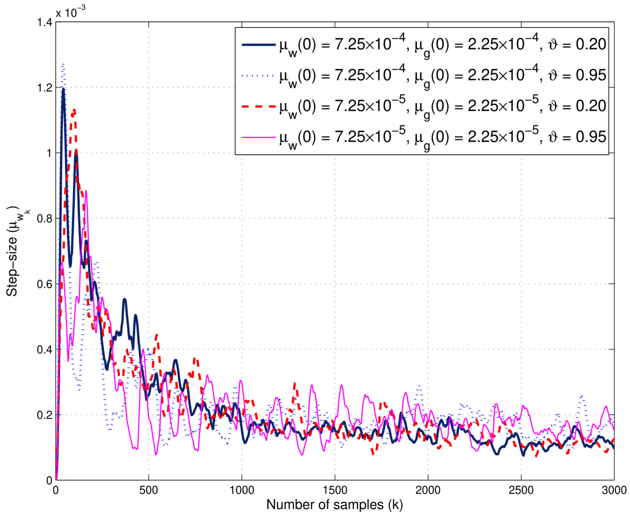

Figure 4: Learning curves of adaptive step-size μ_{w_k} of coefficient vector $\mathbf{w}_k$ of proposed AS-VLLMS-SAF on the *B-spline* matrix $\mathbf{C}_B$ with $\alpha = 0.20, 0.95$ and SNR = 35dB.

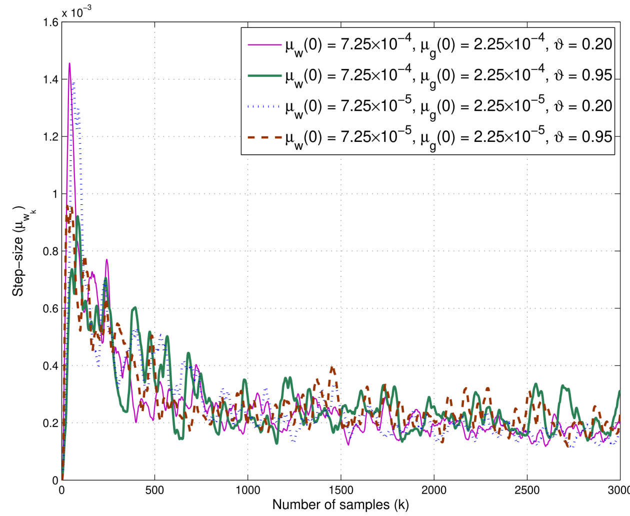

Figure 6: Learning curves of adaptive step-size μ_{w_k} of coefficient vector $\mathbf{w}_k$ of proposed AS-VLLMS-SAF on the *Catmul-Rom* matrix $\mathbf{C}_{CR}$ with $\alpha = 0.20, 0.95$ and SNR = 35dB.

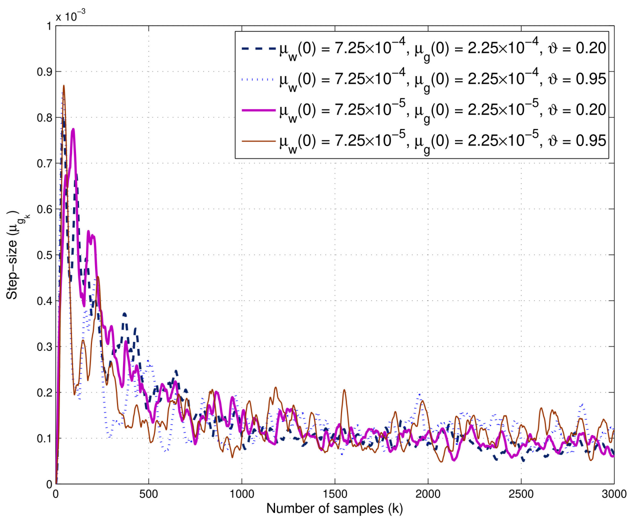

Figure 5: Learning curves of adaptive step-size μ_{g_k} of coefficient vector $\mathbf{g}_{m,k}$ of proposed AS-VLLMS-SAF on the *B-spline* matrix $\mathbf{C}_B$ with $\alpha = 0.20, 0.95$ and SNR = 35dB.

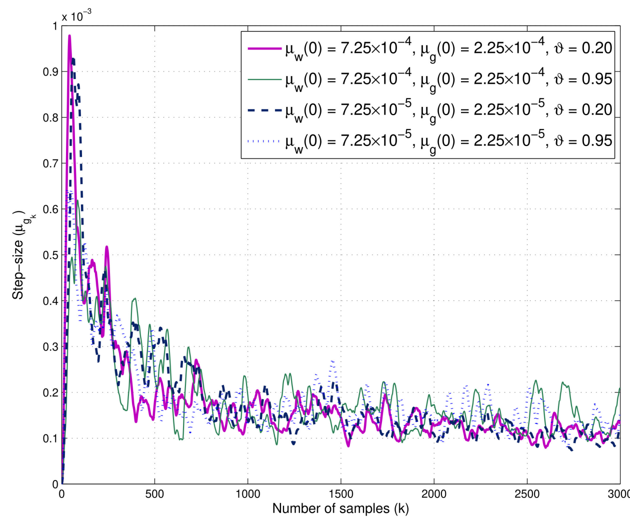

Figure 7: Learning curves of adaptive step-size μ_{g_k} of coefficient vector $\mathbf{g}_{m,k}$ of proposed AS-VLLMS-SAF on the *Catmul-Rom* spline matrix $\mathbf{C}_{CR}$ with $\alpha = 0.20, 0.95$ and SNR = 35dB.

4.2 The Leaky Adjustment

In this section, the leaky adjustment is investigated in form of the *a posterior* error compared with *a priori* error in the spline adaptive filtering. We determine the *a posteriori* LMS error as [1]

$$\tilde{e}_k^{LMS} \triangleq d_k - \mathbf{v}_k^T\,\mathbf{C}_b\,\mathbf{g}_{m,k+1} = e_k\,(\,1 - \mu_g\,\mho_k^T\,\mho_k\,)\,, \qquad (42)$$

where $\mho_k$ is described in (37).

Correspondingly, we examine the *a posteriori* variable leaky least mean square (VL-LMS) error as

$$\begin{aligned}\tilde{e}_k^{VLLMS} &\triangleq d_k - \mathbf{v}_k^T\,\mathbf{C}_b\,\mathbf{g}_{m,k+1}\\ &= e_k(1 - \mu_{g_k}\mho_k^T\,\mho_k) + \mu_{g_k}\gamma_{g_k}\mho_k^T\,\mathbf{g}_{m,k}\,.\end{aligned}$$

It is noticed that the leaky adjustment will be achieve based on a greedy heuristic algorithm with each iteration, which can get the appropriate leaky value, if $|\tilde{e}_k^{VLLMS}| < |\tilde{e}_k^{LMS}|$. That means the proposed VL-LMS algorithm would allow to get the efficient LMS algorithm. Otherwise, the leak parameter should be diminished.

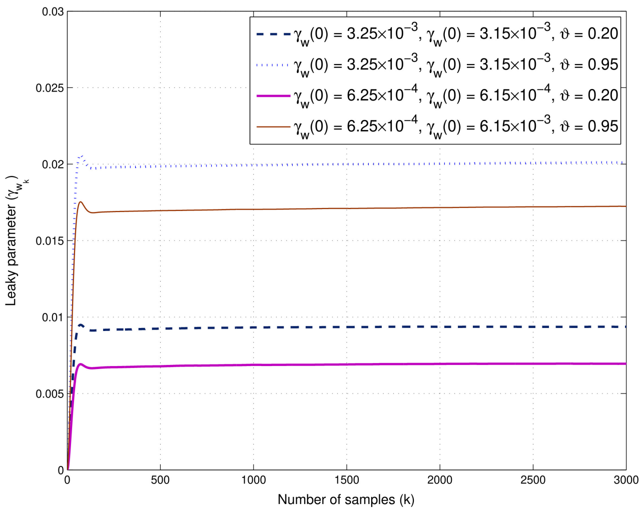

Figure 8: Learning curves of variable leaky γ_{w_k} of coefficient vector $\mathbf{w}_k$ of proposed AS-VLLMS-SAF on $\mathbf{C}_B$ with $\vartheta = 0.20, 0.95$ and SNR = 35dB.

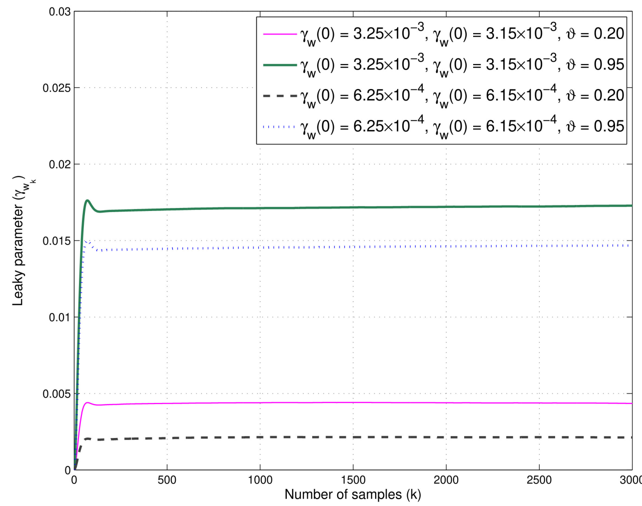

Figure 10: Learning curves of variable leaky γ_{w_k} of coefficient vector $\mathbf{w}_k$ of proposed AS-VLLMS-SAF on $\mathbf{C}_{CR}$ with $\vartheta = 0.20, 0.95$ and SNR = 35dB.

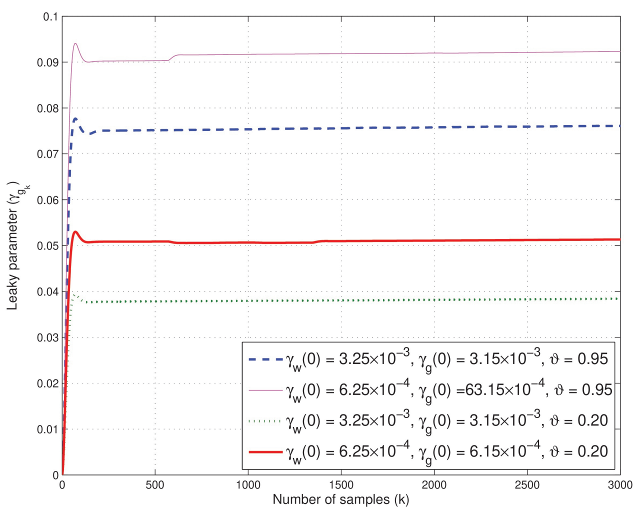

Figure 9: Learning curves of variable leaky γ_{g_k} of coefficient vector $\mathbf{g}_{m,k}$ of proposed AS-VLLMS-SAF on $\mathbf{C}_B$ with $\alpha = 0.20, 0.95$ and SNR = 35dB.

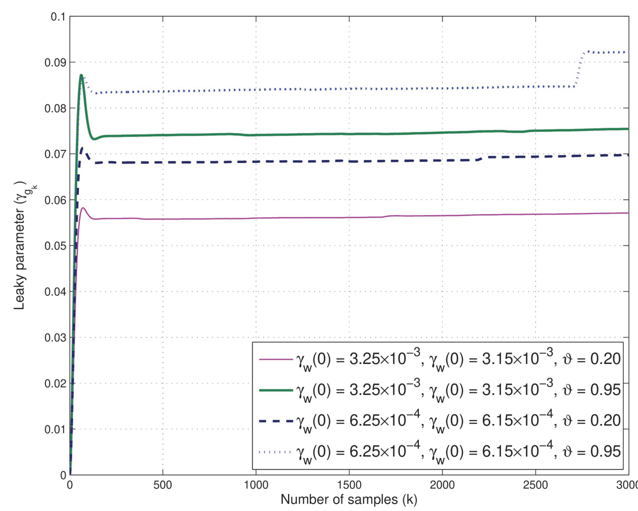

Figure 11: Learning curves of variable leaky γ_{w_k} of coefficient vector $\mathbf{w}_k$ of proposed AS-VLLMS-SAF on $\mathbf{C}_{CR}$ with $\alpha = 0.20, 0.95$ and SNR = 35dB.

4.3 *Mean Square Error Performance*

We investigate the mean square error (MSE) performance of the proposed AS-VLLMS-SAF algorithm at the steady-state.

Assumption 3: *We assume that the a priori and a posteriori optimal errors are identical, we have*

$$E\{\epsilon_g^{\text{opt}}\} \approx e_{\text{opt},g} \ .$$

Assumption 4: *We consider the convergence condition for $k \to \infty$, that is of*

$$E\{\epsilon_g^{\text{opt}}\} \to 0 \ , \ \text{as} \ \ k \to \infty$$
$$E\{\mathbf{g}_{m,k}\} \to \mathbf{g}_{m,k}^{\text{opt}} \ , \ \ \text{as} \ \ k \to \infty$$

Following [12], we decompose the MSE under these assumptions above as follows.

$$\mathbb{J}_{MSE}^{P} = \mathbb{J}_{MMSE}^{P} + \mathbb{J}_{EX}^{P} = E\{\|\epsilon_g\|^2\} \ , \tag{43}$$

where $\mathbb{J}_{MMSE}^{P}$ is the minimum mean square error (MMSE) as

$$\mathbb{J}_{MMSE}^{P} = E\{\|\epsilon_g^{\text{opt}}\|^2\} \ , \tag{44}$$
$$\epsilon_g^{\text{opt}} = d_k - \mathbf{v}_k^T \, \mathbf{C}_b \, \mathbf{g}_{\text{opt}} \ , \tag{45}$$

where ϵ_g^{opt} is the *a posteriori* optimal error of $\mathbf{g}_{\text{opt}}$.

Certainly, $\mathbb{J}_{EX}^{P}$ is the *a posteriori* excess mean square error (EMSE) given by

$$\mathbb{J}_{EX}^{P} = \mathbb{J}_{MSE}^{P} - \mathbb{J}_{MMSE}^{P} = E\{\|\epsilon_g\|^2\} - E\{\|\epsilon_g^{\text{opt}}\|^2\} \ . \tag{46}$$

4.4 Experimental conditions on parameters setting

In this section, the theoretical experiments are simulated in system identification over the random process and under the Gaussian noise scenario. We evaluate the performance of proposed adaptive step-size variable leaky least mean square algorithm based on spline adaptive filter (AS-VLLMS-SAF) as compared to the conventional least mean square algorithm based on spline adaptive filter (LMS-SAF) [3].

The coloured input signal generates for all experiments over 100 Monte Carlo trials and 5,000 samples used that is generated by [2]

$$x_k = \vartheta \cdot x_{k-1} + \sqrt{1-\vartheta^2}\zeta_k \,, \tag{47}$$

where ζ_k is a unitary variance type of zero mean white Gaussian noise, $\vartheta = [0, 0.95]$ and an interval sampling Δx is used at 0.2 [11].

An unknown Wiener system is composed by a linear component as [3]

$$\mathbf{w}_0 = [0.6,\ -0.4,\ 0.25,\ -0.15,\ 0.1]^T \,,$$

and a 23-point length lookup table (LUT) $\mathbf{g}_0$ is a nonlinear memoryless target function applied by

$$\mathbf{g}_0 = [\ -2.2,\ \ldots\ ,-0.8,\ -0.91,\ -0.4,\ -0.2,\ 0.05,\ 0,\ -0.4,\ 1.0,\ 1.0,\ 1.2,\ \ldots\ ,2.2\ ] \,.$$

The linear filter $\mathbf{w}_0$ is initialzed as $\mathbf{w}_0 = [1,\ 0, \ldots,\ 0]$ and $\delta = 0.001$. Other parameters are fixed at the length of coefficient vector $T = 5$, a signal to noise ratio $SNR = 35$dB . The spline basis matrices are used as the *B-spline* matrix $\mathbf{C}_B$ and the *Catmul-Rom spline* matrix $\mathbf{C}_{CR}$ in [8] which are selected for simulation experiments as follows:

$$\mathbf{C}_B = \frac{1}{6}\begin{bmatrix} -1 & 3 & -3 & 1 \\ 3 & -3 & 3 & 0 \\ -3 & 0 & 3 & 0 \\ 1 & 4 & 1 & 0 \end{bmatrix} , \tag{48}$$

$$\mathbf{C}_{CR} = \frac{1}{2}\begin{bmatrix} -1 & 3 & -3 & 1 \\ 2 & -5 & 4 & -1 \\ -1 & 0 & 1 & 0 \\ 0 & 2 & 0 & 0 \end{bmatrix} . \tag{49}$$

The fixed parameters of proposed AS-VLLMS-SAF algorithm are as follows: $\alpha = 0.20, 0.95$, $\alpha_w = 7.55 \times 10^{-3}$, $\beta_w = 2.75 \times 10^{-3}$, $\alpha_g = 6.55 \times 10^{-3}$, $\beta_g = 1.85 \times 10^{-3}$, $\lambda = 0.97$, $\epsilon = 1 \times 10^{-6}$ and $\rho_w = 1.5 \times 10^{-6}$, $\rho_g = 1.125 \times 10^{-6}$. And the initial parameters for proposed AS-VLLMS-SAF are $\gamma_w(0) = 3.25 \times 10^{-2}$, $\gamma_g(0) = 3.25 \times 10^{-2}$, $\mu_w = 7.75 \times 10^{-4}, \mu_g = 2.25 \times 10^{-4}$. Other fixed parameter of SAF-LMS [3] are as: $\mu_w = 0.05$ and $\mu_g = 0.05$.

Table 2: Summary of MSE of proposed AS-VLLMS-SAF with the initial parameters: $\mu_w(0) = 7.25\times10^{-5}$, $\mu_g(0) = 2.25\times10^{-5}$, $\gamma_w(0) = 6.25\times10^{-4}$, $\gamma_g(0) = 6.15\times10^{-4}$, of VL-LMS in [1] with $\gamma_w(0) = 3.15 \times 10^{-4}$, $\gamma_g(0) = 3.15 \times 10^{-4}$ and of LMS-SAF in [3] with $\mu_w = \mu_g = 0.05$ over 100 Monte Carlo trials and 5,000 samples used at SNR = 35dB

Algorithm	ϑ in (47)	Spline matrix	MSE at steady-state condition	
			MSE	dB
AS-VLLMS	$\vartheta = 0.20$	$\mathbf{C}_B$	1.268×10^{-3}	-28.969
		$\mathbf{C}_{CR}$	1.448×10^{-3}	-28.392
	$\vartheta = 0.95$	$\mathbf{C}_B$	1.768×10^{-3}	-29.293
		$\mathbf{C}_{CR}$	2.342×10^{-3}	-26.304
VL-LMS in[1]	$\vartheta = 0.20$	$\mathbf{C}_B$	1.815×10^{-3}	-27.409
		$\mathbf{C}_{CR}$	1.931×10^{-3}	-27.144
	$\vartheta = 0.95$	$\mathbf{C}_B$	2.175×10^{-3}	-26.625
		$\mathbf{C}_{CR}$	2.592×10^{-3}	-25.862
LMS in[3]	$\vartheta = 0.20$	$\mathbf{C}_B$	3.303×10^{-3}	-24.811
		$\mathbf{C}_{CR}$	1.352×10^{-2}	-18.688
	$\vartheta = 0.95$	$\mathbf{C}_B$	8.345×10^{-3}	-20.786
		$\mathbf{C}_{CR}$	2.858×10^{-2}	-15.439

5 Simulation Results

For the experiment results, the mean square error (MSE) is simulated at $\vartheta = 0.20, 0.95$. Fig. 2 and Fig. 3 show the MSE convergence curves of proposed AS-VLLMS-SAF compared with the original LMS-SAF [5] based on the *B-spline* matrix ($\mathbf{C}_B$) matrix and the *Catmul-Rom* spline matrix ($\mathbf{C}_{CR}$) with the parameters $\vartheta = 0.2, 0.95$ in (47) shown in dB and SNR=35dB, respectively. We notice that the curves of MSE of proposed AS-VLLMS-SAF based on both spline basis matrices outperform when comparable to that of the LMS-SAF algorithm.

Furthermore, Fig. 4 and Fig. 5 based on the *B-spline* matrix and Fig. 6 and Fig. 7 based on the *Catmul-Rom* spline matrix demonstrate the curves of learning rate of μ_{w_k} of coefficient vector $\mathbf{w}_k$ and μ_{g_k} of control points $\mathbf{g}_{m,k}$ for the proposed AS-VLLMS-SAF at $\vartheta = 0.20, 0.95$ and SNR=35dB, respectively. Their learning curves are depicted to converge to their equilibrium at the steady-state, even the initial of μ_{w_k} and μ_{g_k} are varied.

Finally, Fig. 8 and Fig. 9 based on the *B-spline* matrix and Fig. 10 and Fig. 11 based on the *Catmul-Rom* spline matrix present the curves of learning parameters of γ_{w_k} of coefficient vector $\mathbf{w}_k$ and γ_{g_k} of adaptive control points vector $\mathbf{g}_{m,k}$ for the proposed AS-VLLMS-SAF at $\vartheta = 0.20, 0.95$ and SNR=35dB, respectively. It can seen clearly that their learning rates are shown to converge for the tracking ability at the steady-state with the different initial parameters.

Summary of MSE of proposed AS-VLLMS-SAF with the initial parameters as $\mu_w(0) = 7.25 \times 10^{-5}$, $\mu_g(0) = 2.25 \times 10^{-5}$, $\gamma_w(0) = 6.25 \times 10^{-4}$, $\gamma_g(0) = 6.15 \times 10^{-4}$ and of LMS-SAF in [3] with the fixed parameter as $\mu_w = \mu_g = 0.05$ over 100 Monte Carlo trials and 5,000 samples used at SNR = 35dB and $\vartheta = 0.20, 0.95$ is presented in Table. 2. Simulation results suggest that mean square error performance of proposed algorithm can be partially assessed using adaptive step-size with the variable leaky parameters indicating better than the conventional least mean square algorithm by 16.76% with the *B-spline* matrix.

6 Discussion

The comparison of MSE averaged over 100 trials are shown the robustness and superiority of proposed AS-VLLMS algorithm over the conventional LMS algorithm. We have plotted the learning curves of adaptive step-size μ_{w_k} of coefficient vector $\mathbf{w}_k$, μ_{g_k} of control points $\mathbf{g}_{m,k}$ for the proposed AS-VLLMS algorithm and of variable leaky γ_{w_k} of coefficient vector $\mathbf{w}_k$ and γ_{g_k} of adaptive control points vector $\mathbf{g}_{m,k}$ after 5,000 iterations, which are seen that their learning curves converges to their equilibrium, even the initial values are assigned to be varied.

7 Conclusion

We have orchestrated an adaptive step-size and a variable leaky approach based on least mean square algorithm for spline adaptive filtering. The proposed AS-VLLMS-SAF algorithm has been explained how to derive using the variable leaky mechanism. We have designed an adaptive step-size algorithm with the methods of an energy of squared previous and present estimated error. We have designed a modified leaky algorithm with the methods of an optimal leaky parameter. Simulation experiments have shown that the proposed AS-VLLMS-SAF algorithm can perform well with the corresponding LMS-SAF algorithm.

In general, spline adaptive filtering structure is already fascinatingly applied in many applications such as signal processing for communications in nonlinear channel estimation and acoustic processing in bio-acoustic signal.

Conflict of Interest The authors declare no conflict of interest.

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