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Travelling Wave Solutions of Coupled Burger's Equations of Time-Space Fractional Order by Novel (G′/G)-Expansion Method

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ABSTRACT

In this paper, Novel (G'/G)-expansion method is used to find new generalized exact travelling wave solutions of fractional order coupled Burger's equations in terms of trigonometric functions, rational functions and hyperbolic functions with arbitrary parameters. For the conversion of the partial differential equation to the ordinary differential equation, complex transformation method is used. Novel (G'/G)-expansion method is very effective and provides a powerful mathematical tool to solve nonlinear equations. Moreover, for the representation of these exact solutions we have plotted graphs for different values of parameters which were in travelling waveform.

1. Introduction

Fractional complex transformation as in [1] is utilized for transformation of nonlinear fractional order partial differential equations into nonlinear ordinary differential equations. The one-dimensional case of Burger's equation was introduced by a Dutch scientist J. M. Burger in 1939 see [2], its general form is given as

$$\partial_t(r) + (r.\nabla)r = s\nabla^2 r.$$

Afterward, new type of Burger's equation was presented in further research named as time-space fractional order coupled Burger's equations [3], which has the form

$$\frac{\partial^\alpha w}{\partial t^\alpha} - \frac{\partial^{2\beta} w}{\partial x^{2\beta}} + 2w\frac{\partial^\beta w}{\partial x^\beta} + c\frac{\partial^\beta (wz)}{\partial x^\beta} = 0,$$

$$\frac{\partial^\alpha z}{\partial t^\alpha} - \frac{\partial^{2\beta} z}{\partial x^{2\beta}} + 2z\frac{\partial^\beta z}{\partial x^\beta} + d\frac{\partial^\beta (wz)}{\partial x^\beta} = 0,$$

where $0 < \alpha \leq 1, 0 < \beta \leq 1, t > 0.$

An assortment of approaches occurs for nonlinear equations which gives their travelling wave and numerical solutions. Wang et al. introduced (G′/G)-expansion method [4]. This method was further reached out for coupled Burger's equations by Younas et al. [3]. In this article Novel (G′/G)-expansion method has been utilized to discover travelling wave solutions of nonlinear space and time fractional order coupled Burger's equations.

1.1. Fractional complex transformation

Fractional complex transform is the unobtrusive way for the transformation of the fractional order differential equation into integer order differential equation. This simplifies the rest of the procedure towards a solution. Presently consider

$$\delta = Lt + Mx + Ny + Oz,$$

where *L, M, N* and *O* are constants.

Let the fractional complex transformation in nonlinear fractional order coupled Burger's equations be defined as

$$\delta = \frac{Lt^\alpha}{\Gamma(\alpha+1)} + \frac{Mx^\beta}{\Gamma(\beta+1)} + \frac{Ny^\gamma}{\Gamma(\gamma+1)} + \frac{Oz^\lambda}{\Gamma(\lambda+1)}.$$

1.2. Travelling wave solution

Travelling wave solution was used for transformation of partial differential equations into ordinary differential equations. For this purpose, we consolidate two independent variables into one independent variable known as travelling wave variable *x* [5].

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Travelling wave solution of nonlinear partial differential equation

$$\partial_t w(z,t) + c\partial_z w(z,t) = 0,$$

where $z \in R,\ t > 0$, can be calculated as follows.

Applying phase plan analysis technique by presenting the travelling wave coordinates as takes after

1. Define the travelling wave variable x as $x = z - bt$, $z \in R$ and $b > 0$ is the propagation speed of the wave or travelling wave speed.
2. Defining the travelling wave Ansatz [6] for $w\,(z,t)$ in the following form
$$W(x) = W(z - dt) = w(z,t)$$

By the travelling wave Ansatz, we have

$$\partial_t w = -d\frac{\partial W}{\partial x} = -dW'(x),$$

$$\partial_z w = \frac{\partial W}{\partial x} = W'(x),$$

$$-bW'(x) + cW'(x) = 0, \quad x = z - bt.$$

1.3. Fractional derivatives in Caputo sense

Fractional derivative presented in the Caputo sense of $f \in C_{-1}^{n}$ [7], is defined as

$$D^\alpha f(r) = \frac{1}{\Gamma(n-\alpha)}\int_0^r (r-\delta)^{n-\alpha-1} f^n(\delta)d\delta.$$

By Caputo's derivative

$$D^\alpha L = 0,$$

where L is constant.

$$D^\alpha (r)^\beta = \frac{\Gamma(\beta+1)}{\Gamma(\beta-\alpha+1)} r^{\beta-\alpha}, \quad \beta > \alpha - 1.$$

Linear relationships of Caputo's derivative are

$$D^\alpha\big(f(r)g(r)\big) = \sum_{n=0}^{\infty}\binom{\alpha}{n} f^{(n)}(r) D^{\alpha-n} f(r),$$

$$D^\alpha\big(\theta f(r) + \varphi g(r)\big) = \theta D^\alpha f(r) + \varphi D^\alpha g(r),$$

where θ and φ are constants. This relation is likewise named as Leibnitz rule.

$$D_t^\alpha w(x,t) = \frac{\partial^\alpha w(x,t)}{\partial t^\alpha},$$

$$= \begin{cases} \dfrac{1}{\Gamma(n-\alpha)}\displaystyle\int_0^t (t-\delta)^{n-\alpha-1}\frac{\partial^n w(x,t)}{\partial t^n}d\delta, & n > \alpha > n-1, \\ \dfrac{\partial^n w(x,t)}{\partial t^n}, & \alpha = n. \end{cases}$$

2. Novel (G′/G)-Expansion Method

Fractional order partial differential equation of the form

$$S(w, w_x, w_t, D_t^\alpha w, \dots) = 0, \tag{1}$$

see [8], where $D_t^\alpha w$ demonstrates the modified form of Riemann-Liouville derivative presented by Jumarie [9], and S is a polynomial of unknown function $w(x,t)$ and its several nonlinear partial and fractional derivatives.

Step I: Fractional complex transformation projected by Li and He [10] was used in order to transform PDE into ODE. For required equations, complex transformation is

$$w(x,t) = w(\delta), \quad \delta = \frac{Lt^\alpha}{\Gamma(\alpha+1)} + \frac{Mx^\beta}{\Gamma(\beta+1)}, \tag{2}$$

where L and M are non-zero arbitrary constants. Complex transformation (2) converts fractional order partial differential (1) into ODE in integer order as

$$S(w, w', w'', w''', \dots) = 0, \tag{3}$$

Step II: Integrate (3) to possible extent. At that time submit the constants which are to be determined later.

Step III: Considering the solution of (3)

$$w(\delta) = \sum_{i=-m}^{m} \alpha_i\big(n + \gamma(\delta)\big)^i, \tag{4}$$

where

$$\gamma(\delta) = \frac{G'(\delta)}{G(\delta)}, \tag{5}$$

α_m and α_{-m} can't be simultaneously zero. n and α_i are constants that will be determined later. $G = G(\delta)$ satisfies the nonlinear ordinary differential equation of second order

$$GG'' = PGG' + QG^2 + R(G')^2, \tag{6}$$

where P, Q and R are genuine constants and the derivative represented by prime is with respect to δ.

By using Cole-Hopf transformation [11, 12], (6) reduces to Riccati equation

$$\gamma'(\delta) = Q + P\gamma(\delta) + (R-1)\gamma^2(\delta). \tag{7}$$

Equation (7) has twenty-five solutions [13].

Step IV: By the homogeneous balance the value of m can be calculated, where $m > 0$.

Step V: Embedding (4) together with (5) and (6) into (3), we get polynomials of the forms $\left(n + \frac{G'}{G}\right)^i$ and $\left(n + \frac{G'}{G}\right)^{-i}$. Assembling each coefficient of polynomials equivalent to zero an overdetermined set is acquired as algebraic equations comprising α_i, L, m and M.

Step VI: Values of α_i , L, m and M the constants can be obtained by solving set of the algebraic equations. Cnsequently the solutions of (6) together with the resulted values of the constants generate exact travelling wave solutions of the nonlinear (1).

3. Application of Novel (G′/G)-Expansion Method

Consider the Coupled Burger's equations of time-space fractional order form as

$$\frac{\partial^\alpha w}{\partial t^\alpha} - \frac{\partial^{2\beta} w}{\partial x^{2\beta}} + 2w\frac{\partial^\beta w}{\partial x^\beta} + c\frac{\partial^\beta (wz)}{\partial x^\beta} = 0, \quad (1)$$

$$\frac{\partial^\alpha z}{\partial t^\alpha} - \frac{\partial^{2\beta} z}{\partial x^{2\beta}} + 2z\frac{\partial^\beta z}{\partial x^\beta} + d\frac{\partial^\beta (wz)}{\partial x^\beta} = 0. \quad (2)$$

By using fractional derivatives, (1) and (2) are converted into ordinary differential equations of integer order. After integration, we get

$$k_1 + Lw - M^2 w' + Mw^2 + cM(wz) = 0, \quad (3)$$

$$k_2 + Lz - M^2 z' + Mz^2 + dM(wz) = 0, \quad (4)$$

here k_1 and k_2 are the constants of integration. By considering now the homogeneous balance of w and w' we get $m = 1$.

The trial solution $w(\delta) = \sum_{i=-m}^{m} \alpha_i \left(n + \left(\frac{G'(\delta)}{G(\delta)}\right)\right)^i$ of Novel (G′/G)-expansion method becomes

$$w(\delta) = \alpha_{-1}\left(n + \frac{G'}{G}\right)^{-1} + \alpha_0\left(n + \frac{G'}{G}\right)^0 + \alpha_1\left(n + \frac{G'}{G}\right)^1. \quad (5)$$

In a similar way, the trial solution for z is

$$z(\delta) = \beta_{-1}\left(n + \frac{G'}{G}\right)^{-1} + \beta_0\left(n + \frac{G'}{G}\right)^0 + \beta_1\left(n + \frac{G'}{G}\right)^1. \quad (6)$$

Using (5) and (6) into (3) also in (4), we obtain polynomials in $\left(n + \frac{G'}{G}\right)^i$ and $\left(n + \frac{G'}{G}\right)^{-i}$ on left-hand side. Now assembling each coefficient of polynomials equal to zero, we get an algebraic set of equations for $\alpha_0, \alpha_{-1}, \alpha_1, \beta_0, \beta_{-1}, \beta_1, k_1, k_2, L$ and M. After solving overdetermined set of the algebraic equations by the use of computational software Maple 18 the required solution sets are

3.1. Set 1

$$P = 0, Q = \frac{3\beta_{-1}(-1 + dc)}{M(-1 + d)}, R = \frac{Mc - M - \alpha_1 + dc\alpha_1}{M(c - 1)},$$

$$L = 0, M = M, n = 0, k_1 = \frac{-4M\beta_{-1}^2(-1 + dc)(c - 1)}{(-1 + d)^2},$$

$$k_2 = \frac{-4M\beta_{-1}^2(-1 + dc)}{(-1 + d)}, \alpha_{-1} = \frac{\beta_{-1}(c - 1)}{-1 + d},$$

$$\beta_1 = \frac{\alpha_1(d - 1)}{c - 1}, \beta_0 = 0, \beta_{-1} = \beta_{-1}, \alpha_1 = \alpha_1, \alpha_0 = 0,$$

where $M, \alpha_{-1}, \alpha_1, \beta_{-1}, \beta_1$ are arbitrary constants.

3.2. Set 2

$$P = \frac{-L\beta_{-1} + 2nL\beta_0 + 2nM{\beta_0}^2 + 2M\beta_{-1}\beta_0}{M^2\beta_{-1}}, M = M, Q = Q,$$

$$R = \frac{-L\beta_0 - M^2\beta_{-1} + {\beta_0}^2 M}{M^2\beta_{-1}}, k_1 = 0, L = L, n = n, \beta_0 = \beta_0,$$

$$k_2 = -Ln\beta_{-1} - Ln^2\beta_0 - M^2 Q\beta_{-1} - M{\beta_{-1}}^2 - Mn^2{\beta_0}^2 - 2Mn\beta_{-1}\beta_0,$$

$$\alpha_{-1} = 0, \alpha_0 = 0, \alpha_1 = 0, \beta_1 = 0, \beta_{-1} = \beta_{-1},$$

where $M, Q, L, n, \beta_0, \beta_1$ are arbitrary constants.

3.3. Set 3

$$P = -\frac{\alpha_0(-1 + d)}{M}, Q = Q, R = 1, L = -M\alpha_0, M = M,$$

$$k_1 = Mn\alpha_{-1}\alpha_0 - M^2 Q\alpha_{-1} + M{\alpha_{-1}}^2 - M{\alpha_{-1}}^2 d - Mnd\alpha_{-1}\alpha_0,$$

$$k_2 = \frac{-M\alpha_{-1}(-2 + d)(-\alpha_0 nc + MQc - 2\alpha_{-1} + d\alpha_{-1} + cd\alpha_{-1} + cdn\alpha_0)}{c^2},$$

$$\beta_{-1} = \frac{\alpha_{-1}(-2 + d)}{c},$$

$$\alpha_{-1} = \alpha_{-1}, \alpha_0 = \alpha_0, \alpha_1 = 0, \beta_0 = 0,$$

$$\beta_1 = 0, \alpha_0 = \alpha_0, n = n,$$

where $Q, M, n, \alpha_0, \alpha_{-1}$ are arbitrary constants.

By using every one of these sets into (5) and (6), then consolidating with the solution $G(\delta)$ of Novel (G′/G)-expansion method, we get travelling wave solutions to Burger's equations.

Now we take *Set 1* and put it in (5) and (6) simultaneously. Then substitute all twenty-five solutions of Novel (G′/G)-expansion method into the resulted equations.

Rewriting *Set 1*, we get values of $\alpha_1, \alpha_{-1}, \beta_{-1}$ and β_1 as

$$\alpha_1 = \frac{M(R - 1)(-1 + c)}{-1 + dc},, \alpha_{-1} = \frac{MQ(-1 + c)}{3(-1 + dc)}, \alpha_0 = 0, n = 0,$$

$$\beta_1 = \frac{M(R - 1)(-1 + d)}{-1 + dc}, \beta_{-1} = \frac{MQ(-1 + d)}{3(-1 + dc)}, P = 0, L = 0.$$

Putting the whole set in (5) and (6) respectively, we get

$$w(\delta)=\alpha_{-1}\left(n+\frac{G'}{G}\right)^{-1}+\alpha_0\left(n+\frac{G'}{G}\right)^{0}+\alpha_1\left(n+\frac{G'}{G}\right)^{1},$$

by putting values of $n,\alpha_{-1},\alpha_1,\beta_{-1}$ and β_1, we get

$$w(\delta)=\frac{MQ(-1+c)}{3(-1+dc)}\left(0+\frac{G'}{G}\right)^{-1}+0\left(0+\frac{G'}{G}\right)^{0}+\frac{M(R-1)(-1+c)}{-1+dc}\left(n+\frac{G'}{G}\right)^{1},$$

$$w(\delta)=\frac{MQ(-1+c)}{3(-1+dc)}\left(\frac{G'}{G}\right)^{-1}+\frac{M(R-1)(-1+c)}{-1+dc}\left(\frac{G'}{G}\right)^{1},$$

Similarly,

$$z(\delta)=\frac{MQ(-1+d)}{3(-1+dc)}\left(\frac{G'}{G}\right)^{-1}+\frac{M(R-1)(-1+d)}{-1+dc}\left(\frac{G'}{G}\right)^{1}$$

When $\rho=P^2-4QR+4Q>0$ and $(R-1)Q\neq 0$ or $(R-1)P\neq 0$ then solutions of $w(\delta)$ are

$$w^1(\delta)=\frac{MQ(c-1)}{3(dc-1)}\left\{-\frac{1}{2(R-1)}\left(P+\sqrt{\rho}tanh(\sqrt{\rho}\delta/2)\right)\right\}^{-1}+\frac{M(R-1)(c-1)}{dc-1}\left\{-\frac{1}{2(R-1)}\left(P+\sqrt{\rho}tanh(\sqrt{\rho}\delta/2)\right)\right\}^{1},$$

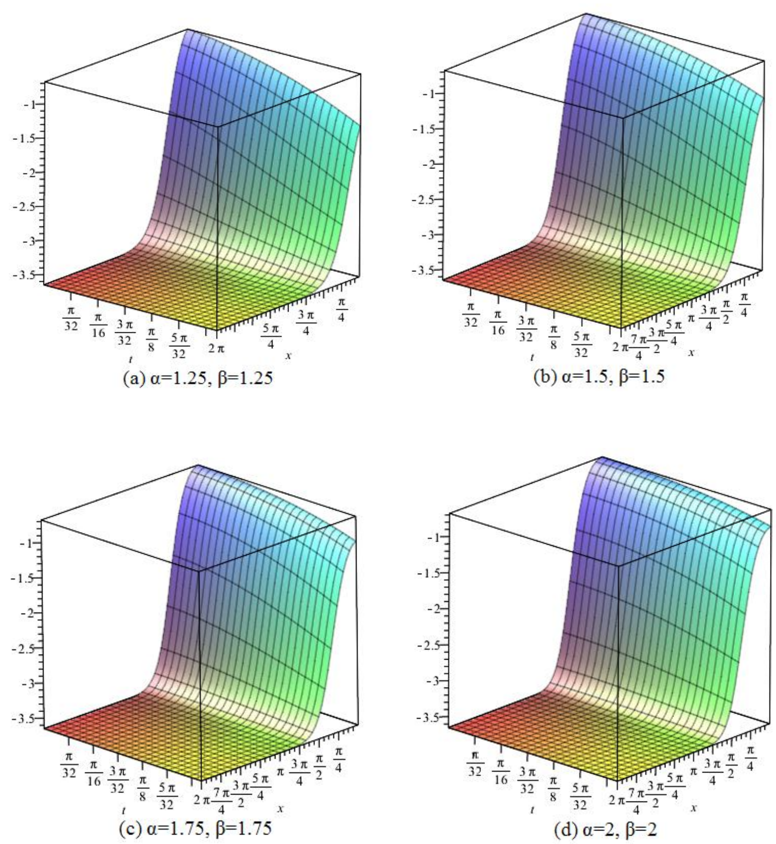

Figure 1: (a) -(d) shows singular soliton solutions of $w^1(\delta)$ for different values of parameters.

$$w^2(\delta) = \frac{MQ(c-1)}{3(dc-1)}\left\{-\frac{1}{2(R-1)}\left(P+\sqrt{\rho}coth(\sqrt{\rho}\delta/2)\right)\right\}^{-1} + \frac{M(R-1)(c-1)}{dc-1}\left\{-\frac{1}{2(R-1)}\left(P+\sqrt{\rho}coth(\sqrt{\rho}\delta/2)\right)\right\}^{1},$$

$$w^3(\delta) = \frac{MQ(c-1)}{3(dc-1)}\left\{-\frac{1}{2(R-1)}\left(P+\sqrt{\rho}\left(tanh(\sqrt{\rho}\delta) \pm isech(\sqrt{\rho}\delta)\right)\right)\right\}^{-1} + \frac{M(R-1)(c-1)}{dc-1}\left\{-\frac{1}{2(R-1)}\left(P+\sqrt{\rho}\left(tanh(\sqrt{\rho}\delta) \pm isech(\sqrt{\rho}\delta)\right)\right)\right\}^{1},$$

$$w^4(\delta) = \frac{MQ(c-1)}{3(dc-1)}\left\{-\frac{1}{2(R-1)}\left(P+\sqrt{\rho}\left(coth(\sqrt{\rho}\delta) \pm icsch(\sqrt{\rho}\delta)\right)\right)\right\}^{-1} + \frac{M(R-1)(c-1)}{dc-1}\left\{-\frac{1}{2(R-1)}\left(P+\sqrt{\rho}\left(coth(\sqrt{\rho}\delta) \pm icsch(\sqrt{\rho}\delta)\right)\right)\right\}^{1},$$

$$w^5(\delta) = \frac{MQ(c-1)}{3(dc-1)}\left\{-\frac{1}{4(R-1)}\left(2P+\sqrt{\rho}\left(tanh(\sqrt{\rho}\delta/4) + icoth(\sqrt{\rho}\delta/4)\right)\right)\right\}^{-1} + \frac{M(R-1)(c-1)}{dc-1}\left\{-\frac{1}{4(R-1)}\left(2P+\sqrt{\rho}\left(tanh(\sqrt{\rho}\delta/4) + icoth(\sqrt{\rho}\delta/4)\right)\right)\right\}^{1}.$$

There are twenty more solutions of w.

Now solutions of $z(\delta)$ when $\rho = P^2 - 4QR + 4Q > 0$ and $(R-1)Q \neq 0$ or $(R-1)P \neq 0$ are

$$z^1(\delta) = \frac{MQ(d-1)}{3(dc-1)}\left\{-\frac{1}{2(R-1)}\left(P+\sqrt{\rho}tanh(\sqrt{\rho}\delta/2)\right)\right\}^{-1} + \frac{M(R-1)(d-1)}{dc-1}\left\{-\frac{1}{2(R-1)}\left(P+\sqrt{\rho}tanh(\sqrt{\rho}\delta/2)\right)\right\}^{1},$$

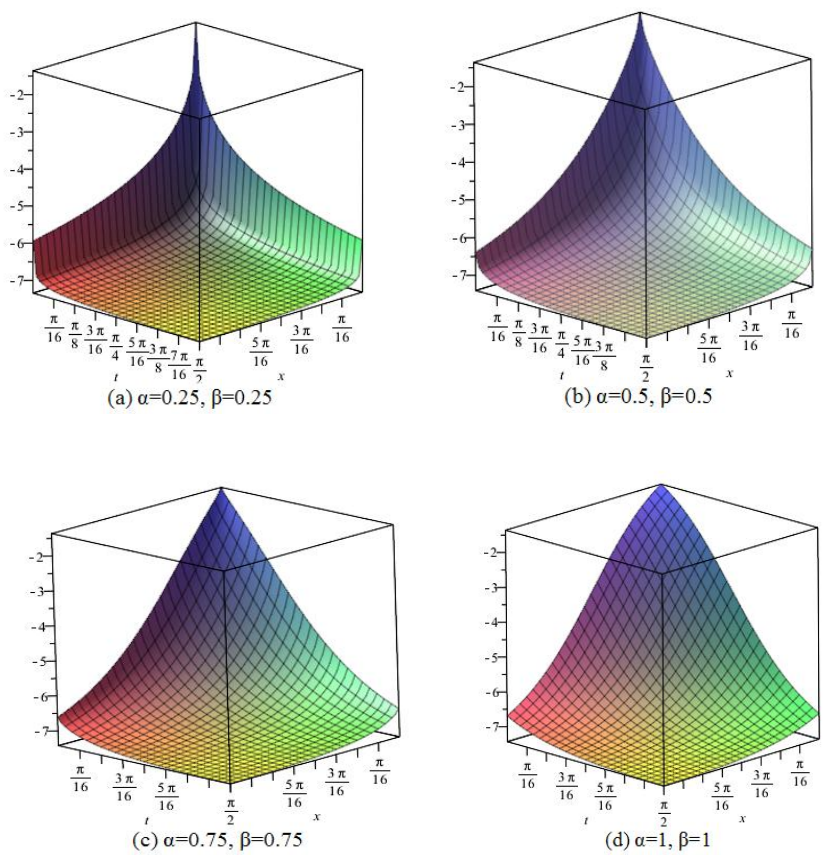

Figure 2: (a) -(d) shows singular soliton solutions of $z^1(\delta)$ for different values of parameters.

$$z^2(\delta)=\frac{MQ(d-1)}{3(dc-1)}\left\{-\frac{1}{2(R-1)}\left(P+\sqrt{\rho}coth\left(\sqrt{\rho}\delta/2\right)\right)\right\}^{-1}+\frac{M(R-1)(d-1)}{dc-1}\left\{-\frac{1}{2(R-1)}\left(P+\sqrt{\rho}coth\left(\sqrt{\rho}\delta/2\right)\right)\right\}^{1},$$

$$z^3(\delta)=\frac{MQ(d-1)}{3(dc-1)}\left\{-\frac{1}{2(R-1)}\left(P+\sqrt{\rho}\left(tanh\left(\sqrt{\rho}\delta\right)\pm isech\left(\sqrt{\rho}\delta\right)\right)\right)\right\}^{-1}+\frac{M(R-1)(d-1)}{dc-1}\left\{-\frac{1}{2(R-1)}\left(P+\sqrt{\rho}\left(tanh\left(\sqrt{\rho}\delta\right)\pm isech\left(\sqrt{\rho}\delta\right)\right)\right)\right\}^{1},$$

$$z^4(\delta)=\frac{MQ(d-1)}{3(dc-1)}\left\{-\frac{1}{2(R-1)}\left(P+\sqrt{\rho}\left(coth\left(\sqrt{\rho}\delta\right)\pm icsch\left(\sqrt{\rho}\delta\right)\right)\right)\right\}^{-1}+\frac{M(R-1)(d-1)}{dc-1}\left\{-\frac{1}{2(R-1)}\left(P+\sqrt{\rho}\left(coth\left(\sqrt{\rho}\delta\right)\pm icsch\left(\sqrt{\rho}\delta\right)\right)\right)\right\}^{1},$$

$$z^5(\delta)=\frac{MQ(d-1)}{3(dc-1)}\left\{-\frac{1}{4(R-1)}\left(2P+\sqrt{\rho}\left(tanh\left(\sqrt{\rho}\delta/4\right)+icoth\left(\sqrt{\rho}\delta/4\right)\right)\right)\right\}^{-1}+\frac{M(R-1)(d-1)}{dc-1}\left\{-\frac{1}{4(R-1)}\left(2P+\sqrt{\rho}\left(tanh\left(\sqrt{\rho}\delta/4\right)+icoth\left(\sqrt{\rho}\delta/4\right)\right)\right)\right\}^{1}.$$

There are twenty more solutions of z.

For convenience, other exact solutions are overlooked.

Conclusion

Novel (G′/G)-expansion method is an effective method for finding exact solutions of the fractional order partial differential equation. As an application, exact solutions have been all around got for time-space fractional order coupled Burger's equations. The fractional complex transformation used as a part of the exhibited work is very momentous. By utilizing this fractional transformation, fractional order partial differential equation can be converted into the integer order ordinary differential equation. Graphical portrayals insure that the required solutions are travelling wave solutions. Novel (G′/G)-expansion method is an influential mathematical tool for solving the nonlinear partial differential equations.

References

[1] J. H. He, S. K. Elagan and Z. B. Li, "Geometrical explanation of the fractional complex transform and derivative chain rule for fractional calculus," Physics Letters. A, **376(4)**: 257–259 (2012).

[2] J. M. Burgers, "Mathematical examples illustrating relations occurring in the theory of turbulent fluid motion," verhand. kon. neder. akad. wetenschappen, afd. natuurkunde, eerste sectie, **17**: 1-53 (1939).

[3] Muhammad Younis, Asim Zafar, Kalim-Ul-Haq and Muhammad Rahman, "Travelling wave solutions of fractional order coupled Burger's equations by (G′/G)-expansion method," American Journal of Computational and Applied Mathematics, **3(2)**: 81 (2013).

[4] M. L. Wang, X. Li and J. Zhang, "The (G′/G)-expansion method and travelling wave solutions of nonlinear evolution equations in Mathematical Physics," Phys. Lett. A, **372**: 417-423 (2008).

[5] Muhamad Abdel Rehim Selim Ibrahim Elshobaki, "Travelling wave solutions of reaction-diffusion equations coupled with shallow water equations," Hamburg, 9-10 (2012).

[6] A. Biswas, M. S. Ismail, "1-Soliton solution of the coupled KdV equation and Gear-Grimshaw model," Appl. Math. Comp., **216**: 36623670 (2010).

[7] Yong Chen and Hong-Li An, "Numerical solutions of coupled Burger's equations with time and space-fractional derivatives," Applied Mathematics and Computation, **200**: 87-95 (2008).

[8] Muhammad Shakeel, Qazi Mehmood Ul-Hussain, Jamshad Ahmad and Tauseef Naqvi, "Exact solutions of the time fractional BBM-Burger's equation by Novel (G′/G)-expansion method," Hindawi Publishing Corporation, Advances in Mathematical Physics, **181594**: 1-15 (2014).

[9] G. Jumarie, "Modified Riemann-Liouville Derivative and Fractional Taylor Series of Non-Differentiable Functions Further Results," Computers and Mathematics with Applications, **51**: 1369 (2006).

[10] Z. Li and J. He, "Fractional complex transform for fractional differential equations," Mathematical and Computational Applications, **15(2)**: 970-973 (2010).

[11] J. D. Cole, "On a quasi-linear parabolic equation occurring in aerodynamics," quart. appl. math., **9**: 225-236 (1951).

[12] E. Hopf, "The partial differential equation, $u_t+uu_x=u_{xx}$," comm. Pure appl. math., **3**: 201-230 (1950).

[13] S. Zhu, "The generalizing Riccati equation mapping method in nonlinear evolution equation: Application to (2+1)-dimensional Boiti-Leon-Pempinelle equation," Chaos, Solitons and Fractals, **37(5)**: 1335-1342 (2008).