

Singular Integral Equations in Electromagnetic Waves Reflection Modeling

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ABSTRACT

The processes of reflection of three-dimensional electromagnetic waves by locally irregular media interfaces are investigated. The problem under study is mathematically reduced to the solution of a boundary value problem for the Maxwell equations in an infinite space with an irregular boundary. In order to develop a numerical algorithm, the potential theory and a special Green's function are applied to reduce the addressed boundary problem to an equivalent system of two hypersingular integral equations. This system is solved with the use of the approximation and collocation method. Special attention is focused on calculation of the kernels of these equations. Results of simulation of the currents induced on the irregularity and reflected field patterns in the resonance frequency range are presented.

1. Introduction

This paper is an extension of the work originally presented at the URSI Commission B International Symposium on Electromagnetic Theory (EMTS 2016) [1]. In this article we model the scattering of three-dimensional electromagnetic waves on a finite impedance section of a wavy surface separating two media. The scattering of electromagnetic waves by a wavy interface between two different media is an important problem for various applications. In this article we investigate the incidence of an arbitrary plane wave on a local inhomogeneity of the interface assuming sufficiently high conductivity of the underlying medium. The mathematical formulation of such problems reduces to that of solving a system of Maxwell equations in non-regular infinite regions. Depending on the polarization of the incident field, the boundary-value problems are reduced to independent systems of hypersingular integral equations solved by specially developed numerical algorithms utilizing a singularity isolating algorithm.

2. Mathematical model and numerical algorithm

2.1. Mathematical formulation of wave scattering problems

Let us assume that an interface between two media is situated in Cartesian coordinates $\square x, y, z$. The interface consists of two half planes Σ that are specified by the equation $y = 0$ and joined by an impedance cylindrical wavy section S that stretches along the z axis. The entire wavy section is located in the half-space $y \geq 0$. The surface S is specified by the equation $y=f(x)$,

$0 \leq x \leq a$, $f(0) = f(a) = 0$, where a is the x length of irregularity S . Let $a = 2\pi$. The reflecting surface $S \cup \Sigma$ does not depend on z coordinate. Denote by D_1 the region over interface $S \cup \Sigma$ where the 3D incident electromagnetic field propagates. This region D_1 is characterized by permittivity ε_1 , permeability μ_1 , and the k_1 wave number $k_1^2 = \omega^2 \varepsilon_1 \mu_1$, where ω is the circular frequency (Figure 1). The time dependence is $\exp(-i\omega t)$.

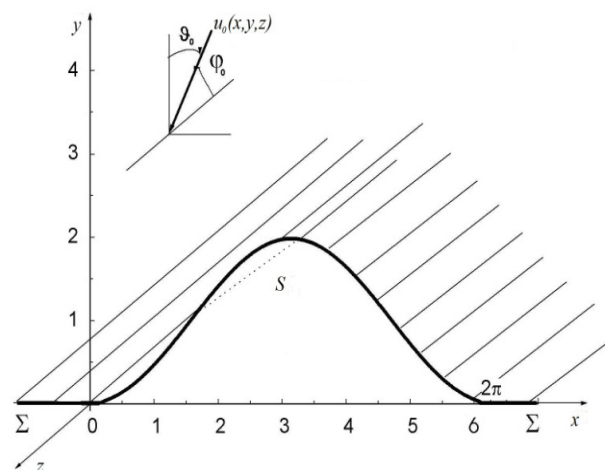


Figure 1: Geometry of perfecting surface

The field of the plane wave

$$\begin{aligned}\vec{E}_0(x, y, z) &= \vec{E}_0 \exp(i\alpha_0 x - i\beta_0 y + i\gamma_0 z), \\ \vec{H}_0(x, y, z) &= \vec{H}_0 \exp(i\alpha_0 x - i\beta_0 y + i\gamma_0 z),\end{aligned}$$

$\alpha_0 = k_1 \sin \vartheta_0 \sin \varphi_0$ \square $\beta_0 = k_1 \cos \vartheta_0 \sin \varphi_0$, and $\gamma_0 = k_1 \cos \varphi_0$ is incident on the surface $S \cup \Sigma$ in region D_1 . Here, φ_0 is the angle between axis z and the direction of the incident wave propagation. When $\varphi_0 = \pi/2$, we obtain a plane reflection two-dimensional problem. If $\vartheta_0 = 0$, we have the case of the normal incidence in the plane problem.

In region D_1 we seek for a solution to the homogeneous system of Maxwell equations

$$\begin{aligned}\text{curl } \vec{H}(x, y, z) + i\omega \varepsilon_1 \vec{E}(x, y, z) &= \vec{0}, \\ \text{curl } \vec{E}(x, y, z) - i\omega \mu_1 \vec{H}(x, y, z) &= \vec{0},\end{aligned}$$

which satisfies on the plane section Σ the condition of perfect conduction

$$[\vec{n} \times \vec{E}(x, y, z)] = \vec{0},$$

Leontovich condition on local section S with a finite conductance

$$[\vec{n} \times \vec{E}(x, y, z)] = -W_2 [\vec{n} \times [\vec{n} \times \vec{H}(x, y, z)]]$$

$W_2 = \sqrt{\mu_2 / \varepsilon_2}$. Here, ε_2 and μ_2 are the characteristics of the section S , $\vec{n} = (n_x, n_y, 0)$ is the outward normal to region D_1 . The reflected field satisfies the radiation conditions at infinity.

We look for the total electromagnetic field in D_1 that has the same dependence on z as the incident field, that is

$$\vec{E}(x, y, z) = \vec{E}(x, y) \exp(i\gamma_0 z), \quad \vec{H}(x, y, z) = \vec{H}(x, y) \exp(i\gamma_0 z).$$

Then, the boundary condition on S can be represented as follows:

$$\begin{aligned}E_z(x, y) &= \frac{W_2}{k_1^2 - \gamma_0^2} \left(-i\gamma_0 \frac{\partial H_z(x, y)}{\partial \tau} - i\omega \varepsilon_1 \frac{\partial E_z(x, y)}{\partial n} \right) \\ H_z(x, y) &= \frac{1}{W_2(k_1^2 - \gamma_0^2)} \left(-i\omega \mu_1 \frac{\partial H_z(x, y)}{\partial n} + i\gamma_0 \frac{\partial E_z(x, y)}{\partial \tau} \right),\end{aligned}$$

$\partial / \partial n = n_x \partial / \partial x + n_y \partial / \partial y$, $\partial / \partial \tau = n_x \partial / \partial y - n_y \partial / \partial x$ are the normal and tangent derivatives, respectively. Accordingly, boundary conditions on plane section Σ take the form

$$E_z(x, y) = 0, \quad \frac{\partial H_z(x, y)}{\partial n} = 0.$$

We denote $u(x, y) = E_z(x, y)$, $v(x, y) = H_z(x, y)$, $\beta_1^2 = k_1^2 - \gamma_0^2$. Then the considered boundary value problem can

be reduced to solving in region D_1 of the 2D homogeneous Helmholtz wave equation

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \beta_1^2 \right) \begin{Bmatrix} u(x, y) \\ v(x, y) \end{Bmatrix} = 0$$

with appropriate boundary conditions on Σ and S .

2.2. Reduction of the boundary problem for Helmholtz equations to the system of singular equations in case of E-polarization

In order to obtain a system of hypersingular integral equations equivalent to the boundary value problem for Helmholtz equation, we introduce Green's function $g^E(M, P)$, satisfying in region D_1 the inhomogeneous Helmholtz equation

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \beta_1^2 \right) g^E(M, P) = -2\pi \delta(M, P),$$

the boundary condition for perfect conductor on half-plane Σ

$$g^E(M, P) = 0,$$

that have the form

$$g^E(M, P) = \frac{i\pi}{2} \left[H_0^{(1)}(\beta_1 r_{M,P}) - H_0^{(1)}(\beta_1 \hat{r}_{M,\hat{P}}) \right].$$

Here, $H_0^{(1)}(x)$ is the zero-order Hankel function of the first kind; $r_{M,P} = \sqrt{(x_M - x_P)^2 + (y_M - y_P)^2}$, where $(x_{M,P}, y_{M,P})$ are coordinates of M and P points; $\hat{r}_{M,\hat{P}} = \sqrt{(x_M - x_P)^2 + (y_M + y_P)^2}$, $(x_P, -y_P)$ are the coordinates of the \hat{P} point.

Applying in region D_1 the Green's formula successively to functions $u(M)$, $g^E(M, P)$, taking into account the radiation conditions and the properties of surface potentials, we obtain a system of two integral equations for unknown functions $\partial u(P) / \partial n$ and $\partial v(P) / \partial \tau$ on finite section S for E-polarization case:

$$\begin{aligned} & \frac{ik_1 W_2}{2W_1 \beta_1^2} \frac{\partial u(M)}{\partial n_M} + \frac{i\gamma_0 W_2}{2\beta_1^2} \frac{\partial v(M)}{\partial \tau_M} + \\ & \frac{1}{2\pi} \int_S \left[\left(g^E(M, P) + \frac{ik_1 W_2}{W_1 \beta_1^2} \frac{\partial g^E(M, P)}{\partial n_P} \right) \frac{\partial u(P)}{\partial n_P} + \right. \\ & \left. \frac{i\gamma_0 W_2}{\beta_1^2} \frac{\partial g^E(M, P)}{\partial n_P} \frac{\partial v(P)}{\partial \tau_P} \right] ds_P = -u_0^E(M), \end{aligned} \quad (1)$$

$$\frac{1}{2} \frac{\partial u(M)}{\partial n_M} - \frac{1}{2\pi} \int_S \left[\left(\frac{\partial g^E(M,P)}{\partial n_M} + \frac{ik_1 W_2}{W_1 \beta_1^2} \frac{\partial^2 g^E(M,P)}{\partial n_M \partial n_P} \right) \frac{\partial u(P)}{\partial n_P} + \frac{i\gamma_0 W_2}{\beta_1^2} \frac{\partial^2 g^E(M,P)}{\partial n_M \partial n_P} \frac{\partial v(P)}{\partial \tau_P} \right] ds_P = \frac{\partial u_0^E(M)}{\partial n_M}, M \in S, \quad (2)$$

where $u_0^E(x, y) = \exp(i\alpha_0 x - i\beta_0 y) - \exp(i\alpha_0 x + i\beta_0 y)$. In the case of the H -polarization we can obtain a system of two integral equations for unknown functions $v(P)$ and $\partial u(P)/\partial \tau$ on finite section S .

2.3. The numerical algorithm for the solution of singular integral equations for the E-polarization

The systems of integral equations are singular and can be solved through reducing them to the complex system of linear algebraic equation by means of the collocation and approximation method.

To this end, we divide the interval $[0, a]$, on which the equation $y = f(x)$ of the irregular cylindrical surface S is defined, by points $x_i = ia/N$, $x_0 = 0$, $x_N = a$, into N equal parts. The points dividing the curve S have the coordinates $(x_i, f(x_i))$. Then we replace the integrals entering into the systems of equations (1), (2) with the sums of the integrals over the subintervals and set $M = M_{i+1/2}$, $i = 1, \dots, N$, $M_{i+1/2}$ are collocation points. The sought solution on the section $S_{P_j, P_{j+1}}$, $j = 0, \dots, N-1$, is approximated by a zero-order spline. As a result, we obtain a complex system of linear algebraic equations. When calculating the integrals over a section $S_{P_j, P_{j+1}}$, in which the integrand depends on either $g^E(M, P)$ or $\frac{\partial g^E(M, P)}{\partial n_P}$, we replace the integration with respect to s with the integration with respect to x by setting $ds_P = \sqrt{1 + (f'(x))^2} dx_P$.

Note that the terms that enter the kernels of integral equations and depend on $g^E(M, P)$ have a logarithmic singularity when the arguments coincide. Therefore, when developing the numerical algorithm, this singularity is separated in an explicit form and the integrals containing it are calculated analytically.

The normal derivatives of the function $g^E(M, P)$ have no singularity when the arguments are equal, and

$$\lim_{P \rightarrow M} \frac{\partial g^E(M, P)}{\partial n_P} = \frac{1}{2} \kappa_0(M),$$

where $\kappa_0(M)$ is the curvature of the contour S at the point M .

The kernels of the system depending on $\frac{\partial^2 g^E(M, P)}{\partial n_M \partial n_P}$ have a strong singularity. In order to calculate the matrix elements that are integrals of the functions with a hypersingularity, it is necessary to develop a numerical algorithm. To this end, we preserve the integration over the arc S in these integrals. We defined the surface S in the form $x = x(s)$, $y = y(s)$. The outward normal to the domain D_1 we write in the form $\vec{n} = -y'_s \vec{i} + x'_s \vec{j}$, where \vec{i} and \vec{j} are the x and y unit vectors. Note that if the boundary of S is specified in the form $y = f(x)$, the relationship between the directional cosines $n_x; n_y$ of the outer normal \vec{n} to D_1 is given by

$$y'_S = -n_x \sqrt{1 + (f'(x))^2}, \quad x'_S = n_y \sqrt{1 + (f'(x))^2}, \\ n_x = f'(x) / \sqrt{1 + (f'(x))^2}, \quad n_y = -1 / \sqrt{1 + (f'(x))^2}.$$

We have

$$\int_{S_{P_j, P_{j+1}}} \frac{\partial^2}{\partial n_M \partial n_P} g^E(M_{i+1/2}, P) ds_P = \quad (3) \\ \frac{i\pi}{2} \int_{S_{P_j, P_{j+1}}} \frac{\partial^2}{\partial n_M \partial n_P} H_0^{(1)}(\beta_1 r_{M_{i+1/2}, P}) ds_P - \\ \frac{i\pi}{2} \int_{S_{P_j, P_{j+1}}} \frac{\partial^2}{\partial n_M \partial n_P} H_0^{(1)}(\beta_1 \hat{r}_{M_{i+1/2}, P}) ds_P.$$

To develop a special computing algorithm, every integrands is represented by a sum of two terms: the first one is the total differential of some function has been calculated analytically; the function in the second term has a logarithmic singularity and is integrated by separating this singularity in the explicit form, thereafter the corresponding integrals are calculated analytically.

In the first term $\partial / \partial n_M$ let us consider the component $\partial / \partial x_M$ (without the multiplier $i\pi/2$):

$$\int_{S_{P_j, P_{j+1}}} \frac{\partial^2}{\partial x_M \partial n_P} H_0^{(1)}(\beta_1 r_{M_{i+1/2}, P}) ds_P = \\ - \int_{S_{P_j, P_{j+1}}} \frac{\partial}{\partial n_P} \frac{\partial}{\partial x_P} H_0^{(1)}(\beta_1 r_{M_{i+1/2}, P}) ds_P = \\ - \int_{S_{P_j, P_{j+1}}} d \left(\frac{\partial}{\partial y_P} H_0^{(1)}(\beta_1 r_{M_{i+1/2}, P}) \right) -$$

$$\begin{aligned}
 & \beta_1^2 \int_{S_{P_j, P_{j+1}}} H_0^{(1)}(\beta_1 r_{M_{i+\frac{1}{2}}, P}) y'_{S_P} ds_P + \\
 & 4i \int_{S_{P_j, P_{j+1}}} \delta(M, P) y'_{S_P} ds_P = \\
 & -\frac{\partial}{\partial y_P} H_0^{(1)}(\beta_1 r_{M_{i+\frac{1}{2}}, P_{j+1}}) + \frac{\partial}{\partial y_P} H_0^{(1)}(\beta_1 r_{M_{i+\frac{1}{2}}, P_j}) - \\
 & \beta_1^2 \int_{S_{P_j, P_{j+1}}} H_0^{(1)}(\beta_1 r_{M_{i+\frac{1}{2}}, P}) y'_{S_P} ds_P + \\
 & 4i \int_{S_{P_j, P_{j+1}}} \delta(M, P) y'_{S_P} ds_P.
 \end{aligned} \tag{4}$$

The points M and P can coincide, hence, there is an integral of the Dirac delta-function.

Similarly let us consider the component $\partial / \partial y_M$ in the first term $\partial / \partial n_M$:

$$\begin{aligned}
 & \int_{S_{P_j, P_{j+1}}} \frac{\partial^2}{\partial y_M \partial n_P} H_0^{(1)}(\beta_1 r_{M_{i+\frac{1}{2}}, P}) ds_P = \\
 & - \int_{S_{P_j, P_{j+1}}} \frac{\partial}{\partial n_P} \frac{\partial}{\partial y_P} H_0^{(1)}(\beta_1 r_{M_{i+\frac{1}{2}}, P}) ds_P = \\
 & \int_{S_{P_j, P_{j+1}}} d \left(\frac{\partial}{\partial x_P} H_0^{(1)}(\beta_1 r_{M_{i+\frac{1}{2}}, P}) \right) + \\
 & \beta_1^2 \int_{S_{P_j, P_{j+1}}} H_0^{(1)}(\beta_1 r_{M_{i+\frac{1}{2}}, P}) x'_{S_P} ds_P - \\
 & 4i \int_{S_{P_j, P_{j+1}}} \delta(M, P) x'_{S_P} ds_P = \\
 & \frac{\partial}{\partial x_P} H_0^{(1)}(\beta_1 r_{M_{i+\frac{1}{2}}, P_{j+1}}) - \frac{\partial}{\partial x_P} H_0^{(1)}(\beta_1 r_{M_{i+\frac{1}{2}}, P_j}) + \\
 & \beta_1^2 \int_{S_{P_j, P_{j+1}}} H_0^{(1)}(\beta_1 r_{M_{i+\frac{1}{2}}, P}) x'_{S_P} ds_P - \\
 & 4i \int_{S_{P_j, P_{j+1}}} \delta(M, P) x'_{S_P} ds_P.
 \end{aligned} \tag{5}$$

Using (4)-(5) and the recurrence relations for Hankel functions

$$\frac{\partial}{\partial x_P} H_0^{(1)}(\beta_1 r_{M, P}) = \beta_1 H_1^{(1)}(\beta_1 r_{M, P}) \frac{x_M - x_P}{r_{M, P}};$$

$$\frac{\partial}{\partial y_P} H_0^{(1)}(\beta_1 r_{M, P}) = \beta_1 H_1^{(1)}(\beta_1 r_{M, P}) \frac{y_M - y_P}{r_{M, P}},$$

we obtain the final formula for the calculation of the second normal derivative of the Green function [2].

2.4. Calculation of the reflected field pattern for the E-polarization

Using the solution to the system of integral equations (1)-(2) and taking into account the boundary conditions on the section S , we can obtain the integral representation for the field $u_1(M) = u(M) - u_0^E(M)$:

$$\begin{aligned}
 u_1(M) = \frac{1}{2\pi} \int_S \left\{ \left[g^E(M, P) + \frac{i k_1 W_2}{\beta_1^2 W_1} \frac{\partial g^E(M, P)}{\partial n_P} \right] \frac{\partial u(P)}{\partial n_P} + \right. \\
 \left. \frac{i \gamma_0 W_2}{\beta_1^2} \frac{\partial g^E(M, P)}{\partial n_P} \frac{\partial v(P)}{\partial \tau_P} \right\} ds_P, \quad M \in D_1. \tag{6}
 \end{aligned}$$

We introduce spherical coordinate frame $r, \bar{\vartheta}, \bar{\varphi}$, where angle $\bar{\vartheta}$ ($-\pi/2 < \bar{\vartheta} < \pi/2$) is measured from the y axis and angle $\bar{\varphi}$ ($0 < \bar{\varphi} < \pi/2$) is measured from the z axis. We seek for a representation of the scattered field for $|\bar{r}|_M \gg |\bar{r}|_P, k_1 |\bar{r}|_M \gg 1$.

Following that, we introduce the notation $|\bar{r}|_M = \rho$. It can be shown that the following asymptotic relationships $r_{MP} \approx \rho - (\bar{r}_P, \bar{e}_M)$ and $\hat{r}_{M\hat{P}} \approx \rho - (\hat{r}_P, \bar{e}_M)$ are valid. Here, \bar{e}_M is a unit vector oriented in the direction to point M ; i.e. $\bar{e}_M = \bar{r}_M / \rho = (\sin \bar{\vartheta} \sin \bar{\varphi}, \cos \bar{\vartheta} \sin \bar{\varphi}, \cos \bar{\varphi})$. Taking into account the above relationships and asymptotic behavior of the Hankel function, we can obtain the asymptotic representations for $g^E(M, P), \frac{\partial g^E(M, P)}{\partial n_P}$ in the form

$$\begin{aligned}
 g^E(M, P) \approx \sqrt{\frac{\pi}{2\beta_1 \rho}} \exp\left(i\left(\beta_1 \rho + \frac{\pi}{4}\right)\right) \times \\
 \left[\exp(-i\beta_1(\bar{r}_P, \bar{e}_M)) - \exp(-i\beta_1(\hat{r}_P, \bar{e}_M)) \right]; \\
 \frac{\partial g^E(M, P)}{\partial n_P} \approx \sqrt{\frac{\pi}{2\beta_1 \rho}} \exp\left(i\left(\beta_1 \rho + \frac{\pi}{4}\right)\right) \times \\
 \left[-i\beta_1(\bar{n}_P, \bar{e}_M) \exp(-i\beta_1(\bar{r}_P, \bar{e}_M)) + \right. \\
 \left. i\beta_1(\bar{n}_{\hat{P}}, \bar{e}_M) \exp(-i\beta_1(\hat{r}_P, \bar{e}_M)) \right] \tag{7}
 \end{aligned}$$

and also for $u_1(M)$ [3].

We call quantity $F^E(\bar{\vartheta}, \bar{\varphi})$ that equals

$$F^E(\bar{\vartheta}, \bar{\varphi}) = \frac{1}{\sqrt{\beta_1}} \int_S \left\{ \left[\exp(-i\beta_1(\bar{r}_P, \bar{e}_M)) - \exp(-i\beta_1(\bar{r}_{\hat{P}}, \bar{e}_M)) \right] + \frac{k_1 W_2}{\beta_1 W_1} \left[(\bar{n}_P, \bar{e}_M) \exp(-i\beta_1(\bar{r}_P, \bar{e}_M)) - (\bar{n}_{\hat{P}}, \bar{e}_M) \exp(-i\beta_1(\bar{r}_{\hat{P}}, \bar{e}_M)) \right] \frac{\partial u(P)}{\partial n_P} + \frac{\gamma_0 W_2}{\beta_1} \left[(\bar{n}_P, \bar{e}_M) \exp(-i\beta_1(\bar{r}_P, \bar{e}_M)) - (\bar{n}_{\hat{P}}, \bar{e}_M) \exp(-i\beta_1(\bar{r}_{\hat{P}}, \bar{e}_M)) \right] \frac{\partial v(P)}{\partial \tau_P} \right\} ds_P$$

the pattern of scattering field $u_1(M)$. Knowing the solution of the system of singular integral equations (1)-(2), we calculate the currents induced on the irregular surface and far-field pattern for the scattered field in the far zone [4].

3. Numerical results

The induced currents and reflected electromagnetic field patterns are calculated for E - polarization, $\epsilon_2 = 50 + 4i$, contour S specified in the XOY plane by the equation $f(x) = 1 - \cos x$, $0 \leq x \leq 2\pi$, $a = 2\pi$.

Figure 2 shows the distribution of the calculated function of the induced current $I(x) = |\partial u(P) / \partial n|$ on the boundary S along the x axis for $ak_1 = 3\pi$, $a/\lambda = 1.5$, where λ is the wavelength of the incident field. The results are obtained for various angles ϑ_0 and φ_0 of the electromagnetic wave incidence: (curve 1) $\vartheta_0 = 0^\circ$, $\varphi_0 = 85^\circ$; (curve 2) $\vartheta_0 = 30^\circ$, $\varphi_0 = 60^\circ$; (curve 3) $\vartheta_0 = 45^\circ$, $\varphi_0 = 45^\circ$; (curve 4) $\vartheta_0 = 60^\circ$, $\varphi_0 = 30^\circ$.

Figures 3-4 show for $ak_1 = 3\pi$, $a/\lambda = 1.5$ reflected field pattern $|F^E(\bar{\vartheta}, \bar{\varphi})|$ in polar coordinates for the range $-85^\circ < \bar{\vartheta} < 85^\circ$ and fixed values of $\bar{\varphi}$. Curves 1-5 correspond to $\bar{\varphi} = 85^\circ, 60^\circ, 45^\circ, 30^\circ, 5^\circ$. Figures 3-4 represent the results of calculations for the reflected field when a plane wave incident at the angles $\vartheta_0 = 0^\circ, \varphi_0 = 85^\circ$ (Figure 3) and $\vartheta_0 = 45^\circ, \varphi_0 = 45^\circ$ (Figure 4).

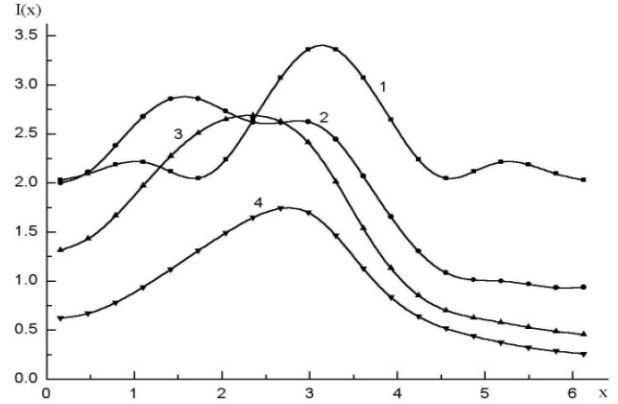


Figure 2

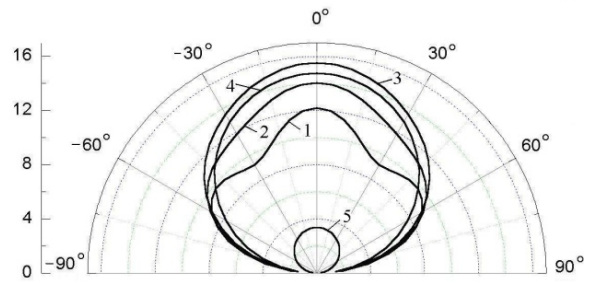


Figure 3

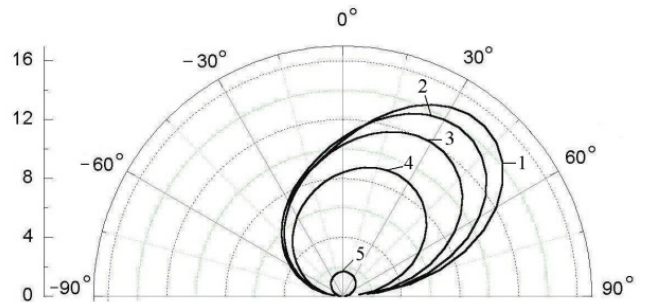


Figure 4

Further, the analogous results are presented on Figures 5-7 for $ak_1 = 5\pi$, $a/\lambda = 2.5$.

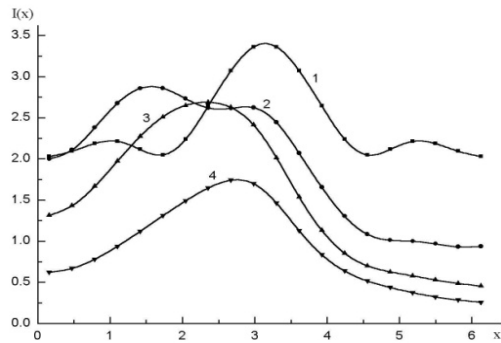


Figure 5

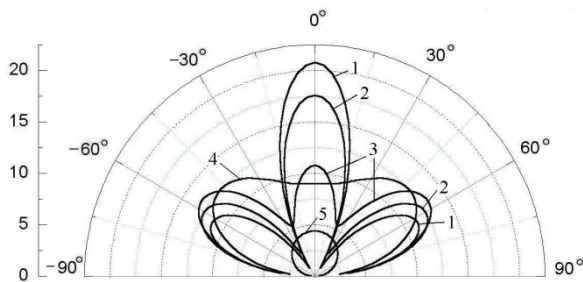


Figure 6

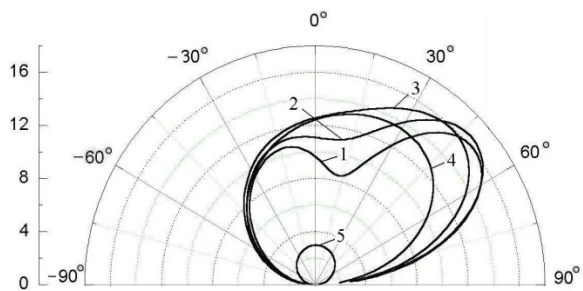


Figure 7

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4. Conclusion

1. The exact model of plane electromagnetic wave incident on inhomogeneous conductive surface is investigated.
2. The problem is reduced to the system of singular integral equations. The analytic solution is provided for a singularity in the kernels of integral equations.
3. The numerical algorithm for calculation of scattering characteristics of incident electromagnetic field was developed and calculation examples were presented.